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Electron acoustic solitary waves in the Earth's magnetotail region

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Abstract

Satellite observations have revealed solitary potential structures in the Earth's magnetotail region. These structures have both positive (compressive) and negative (rarefactive) electrostatic potentials. In this paper we study the electron-acoustic solitary waves (EASWs) in an unmagnetized plasma consisting of cold plasma electrons and isothermal ions with two different temperatures. Using the reductive perturbation method, the nonlinear evolution of such structures is studied. The numerical computations are performed to study the role of two temperature ions in the generation of EASWs. In this case, the model supports the existence of both positive and negative electrostatic potentials with bipolar pulses. The electric field associated with these positive and negative solitary structures are numerically computed. The present study could be useful to construe the compressive and rarefactive electric field bipolar pulses associated with the BEN type emissions in the magnetospheric regions where the electron beams are not present.

Keywords: Electron acoustic solitary waves; Electric field bipolar pulses; Broadband electrostatic noise

1. Introduction

There have been numerous observations of small temporal and spatial scale, large amplitude electric fields in the Earth's magnetotail region. These structures have been commonly called solitary waves or weak doubles layers and appear to be prevalent throughout many parts of the Earth's magnetosphere. For instance, they have been reported in the plasma sheet boundary layer (PSBL) by the Geotail satellite (Matsumoto et al., 1994), by the Plasma Wave Instrument on the Polar satellite (Franz et al., 1998; Tsurutani et al., 1998), and by the Electric Field Instrument (EFI) on the Polar satellite (Cattell et al., 1999). It is well known that the Broadband Electrostatic Noise (BEN) is a common wave activity in the plasma sheet boundary layer region. These type of emissions consist of a series of isolated bipolar pulses. They are termed after their waveforms as electrostatic solitary waves (ESWs). The frequency of these BENs range from ~ 10 Hz upto the local electron plasma frequency (~ 10 kHz) and the electric field intensities are of the order of $\sim$ a few mV/m. The time scale of these ESWs suggests that they are related to electron dynamics rather than ions (Matsumoto et al., 1994). Onsager et al. (1993) described the correlation of BEN with the high energy electron component in the absence of ion flows. It is also found that these solitary waves in the PSBL are either an electron hole or electron acoustic solitary waves propagating with velocities of a few 1000 km/s (Cattell et al., 1999).

An electron-acoustic wave can either exist in a two temperature (cold and hot) electron plasma (Watanabe and Taniuti, 1977) or in an electron ion plasma with ions hotter than electrons (Fried and Gould, 1961). In multi-component plasma, several theoretical efforts have been made to explain these electric field structures in terms of linear (Singh and Lakhina, 2001) and nonlinear electron acoustic instabilities (Singh et al., 2001 and references their in), especially for the high-frequency components. Also, attempts have been made to explain the space observations

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(e.g. Viking) as being solitary electron acoustic structures with negative potential solitary waves in two temperature electron plasmas (e.g. Dubouloz et al., 1991; Mace et al., 1991; Singh and Lakhina, 2004; Tagare et al., 2004). These models could not explain the solitary structures with a positive potential. To explain the generation mechanism of these positive potential structures, recently, Verheest et al. (2005) have pointed out the possibility to obtain compressive electron acoustic solitons if the hot electron inertia is retained in the analysis. Also a few recent theories suggested that the presence of electron beams along with two temperature (low and high) electron plasmas can develop positive potential structures. These models show that depending on the electron beam density and temperature and below a critical velocity of the electron beam, these nonlinear structures can have a positive potential (Berthomier et al., 2000; El-Taibany, 2005).

The spacecraft observations, in the Earth's plasma sheet boundary layer, show the existence of background electrons and the cold electron beams having energies of the order of a few eV to a few 100s of eV and background cold ions and warm ions/ion beams with energies from a few keV to 10s of keV (Parks et al., 1984; Onsager et al., 1993). In view of this, (Kakad et al., 2007) studied the nonlinear evolution of large amplitude electron-acoustic solitary waves (EASWs) in a 4-component plasma system (i.e. cold electron, cold electron beam, low and high temperature ions). In this paper, using the reductive perturbation method, we have studied the effect of two temperature ions on the nonlinear evolution of small amplitude EASWs for the 3-component plasma (i.e. cold electron, low and high temperature ions). This approach leads to a Korteweg-de Vries (KdV) equation which describes the evolution of small amplitudes EASWs. An analytical expression for the potential of the nonlinear structures can be derived from the KdV equation.

The paper is organized as follows: We present the basic set of equations of the model in Section 2. In Section 3, we describe the theory for small amplitude EASWs. In Section 4, we performed numerical computations and describe the results. Finally in Section 5, the application of the results to the electrostatic solitary structures observed in the Earth's plasma sheet boundary layer is discussed.

2. The governing equations

We consider an infinite, collisionless and unmagnetized plasma with three components, namely, cold plasma electrons and isothermal ions with two different temperatures obeying Boltzmann type distributions. The normalized fluid equations for the cold plasma electrons is given by

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e u_e)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = \frac{\partial \phi}{\partial x} \tag{2}$$

Here, n_e and u_e are plasma density and velocity of the cold electrons, and ϕ is the electrostatic potential. Furthermore system of equations is closed by Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_{il} - n_{ih} \tag{3}$$

where

$$n_{il} = \mu e^{-\frac{\phi}{\mu + \nu\beta}}, n_{ih} = \nu e^{-\frac{\beta\phi}{\mu + \nu\beta}} \tag{4}$$

are the densities of the isothermal ions with two temperatures, one species with low temperature T_l and initial normalized equilibrium density μ and the second ion species with high temperature T_h and initial normalized equilibrium density ν. Here $\mu + \nu = 1, \beta = \frac{T_l}{T_h}$ and $T_{eff} = \frac{T_l T_h}{\mu T_h + \nu T_l}$. The subscripts e, l, h refer to cold plasma electrons, low-temperature ions and high-temperature ions. In the above set of equations, the plasma densities, plasma velocities, electrostatic potential, distance x and time t are normalized to the unperturbed total electron plasma density n_0, effective electron acoustic speed $C_{eff} = \left(\frac{T_{eff}}{m_e}\right)^{1/2}$, effective electrostatic potential $\frac{T_{eff}}{e}$, effective Debye length $\lambda_{Deff} = \left(\frac{T_{eff}}{4\pi n_0 e^2}\right)^{1/2}$ and inverse of electron plasma frequency ω_{pe}^{-1}, $\left(\omega_{pe} = \sqrt{\frac{4\pi n_0 e^2}{m_e}}\right)$, respectively.

A linear dispersion relation for electron-acoustic waves can be obtained by linearizing and solving a set of Eqs. (1)–(4) and is given by

$$\omega^2 = \omega_{pe}^2 \left[\frac{k^2 \lambda_{Deff}^2}{1 + k^2 \lambda_{Deff}^2}\right] \tag{5}$$

where ω_{pe} is the cold electron plasma frequency. For the limit $k\lambda_{Deff} \ll 1$ Eq. (5) reduces to $\omega = kC_{eff}$, which is similar to the dispersion relation for the electron-acoustic modes derived by Fried and Gould (1961) for the case of electron-ion plasma with hot ions; i.e. $T_i \gg T_e$.

3. Theory for small amplitude EASWs

To solve the Eqs. (1)–(4) for weakly nonlinear EASWs solutions, we shall use the reductive perturbation method (Washimi and Taniuti, 1966) by introducing the stretched coordinates

$$\xi = \epsilon^{\frac{1}{2}}(x - Vt), \quad \tau = \epsilon^{\frac{3}{2}} t \tag{6}$$

where V is the phase velocity of the wave normalized to electron-acoustic velocity and ϵ is a small parameter measuring the strength of nonlinearity. We expand the dependent variables of Eqs. (1)–(4) around the equilibrium state in power series of ϵ as:

$$n_e = 1 + \epsilon n_{e1} + \epsilon^2 n_{e2} + \dots \tag{7}$$

$$u_e = \epsilon u_{e1} + \epsilon^2 u_{e2} + \dots \tag{8}$$

and

$$\phi = \epsilon\phi_1 + \epsilon^2\phi_2 + \dots \tag{9}$$

Substituting these expansions and stretched variables (ξ and t) from Eq. (6) into the basic set of Eqs. (1)–(4), and then collecting terms with different powers of ϵ, we get the wave velocity by solving the set of equations and comparing the terms of the order of ϵ

$$V = \pm 1 \tag{10}$$

Proceeding to order of ϵ^2, and by eliminating the second order quantities we get the usual modified Korteweg-de Vries (KdV) equation

$$\frac{\partial\phi_1}{\partial\tau} + A\phi_1\frac{\partial\phi_1}{\partial\xi} + \frac{V^3}{2}\frac{\partial^3\phi_1}{\partial\xi^3} = 0 \tag{11}$$

where

$$A = \frac{V^3}{2}\left[\frac{\mu + \nu\beta^2}{(\mu + \nu\beta)^2} - \frac{3}{V^4}\right] \tag{12}$$

A stationary solitary wave solution of the KdV Eq. (11) can be obtained by transforming the space variable by substituting $\chi = \xi - \delta M\tau$ and by imposing the boundary conditions for localized perturbations, viz. $\phi_1(\chi) \to 0$, $d\phi_1/d\xi \to 0$, $d^2\phi_1/d\xi^2 \to 0$, as $|\chi| \to \infty$. Thus, the steady state solution of (11) can be expressed as (Tagare et al., 2004)

$$\phi_1 = A_1 sech^2\left(\frac{\chi}{\Lambda}\right) \tag{13}$$

where A_1 is the amplitude and Λ is the width of the electron-acoustic soliton in a three component plasma system. Furthermore, the parameters A_1 and Λ are given by

$$A_1 = \frac{3\delta M}{A} \tag{14}$$

and

$$\Lambda = \sqrt{\frac{2V^3}{\delta M}} \tag{15}$$

Here δM is an arbitrary parameter similar to Mach number which allows the possibility of solitons moving with velocities different from the phase velocity of the wave (Tagare et al., 2004). Eq. (13) describes the solution of small amplitude electron acoustic wave.

The existence of compressive or rarefactive solitons depends on the sign of nonlinear coefficient A of the KdV Eq. (11). The necessary condition for the compressive (rarefactive) solitons to exist is when $A > 0 (< 0)$. From Eq. (12), it is clear that mathematically the sign of A can be either positive or negative. Further, it is seen from Eq. (12) that the ion temperature ratio and the low temperature ion density decide the regime of positive and negative values of A, and hence the parametric regime of compressive or rarefactive electron acoustic solitons.

4. Numerical results

We compute the amplitudes of EASWs from Eq. (13) numerically for the Earth's plasma sheet boundary layer region parameters (Parks et al., 1984; Onsager et al., 1993; Matsumoto et al., 1994; Omura et al., 1999; Singh and Lakhina, 2001). Fig. 1 shows the variation of amplitude ϕ_1 with χ for different low temperature ion density, μ. It is seen that the low temperature ion density, and the ion temperature ratios (β) play an important role in changing the polarity of small amplitude EASWs. If we consider $\mu = 0$, then this model retrieves only the rarefactive EASWs (not shown in figure). This indicates that the presence of two temperature ions is a necessary condition for the generation of compressive EASWs. For the parameter regime considered here, we find that for $\mu < 0.25$ this model supports compressive potential structures whereas for $\mu > 0.25$ it supports the rarefactive potential structures. In case of compressive structures the maximum amplitude and the width of the EASWs increases with the increase of low temperature ion density. Whereas in case of rarefactive potential structures the maximum amplitude and the width of EASWs decreases with increase in low temperature ion density μ. In this case the associated electric field structures of both the compressive and rarefactive EASWs are found to be bipolar. These structures are shown in Fig. 2 for different low temperature ion density μ.

In Fig. 3 we have shown the variation of potential, ϕ_1 with χ, for different ion temperature ratio β. It is found that for $0 \leqslant \beta < 0.07$ this model supports the compressive EASWs, whereas for $\beta > 0.07$ the polarity of the EASWs poten-

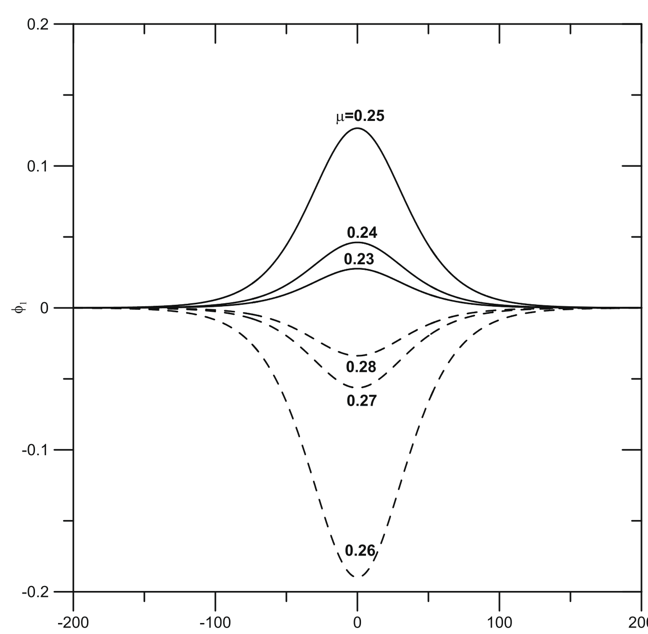

Fig. 1. Variation of potential ϕ_1 with χ for different low temperature ion density, μ. Other parameters are: $\beta = 0.05$, $\delta M = 0.001$. The compressive modes are shown by (—) and rarefactive mode are shown by (- - -).

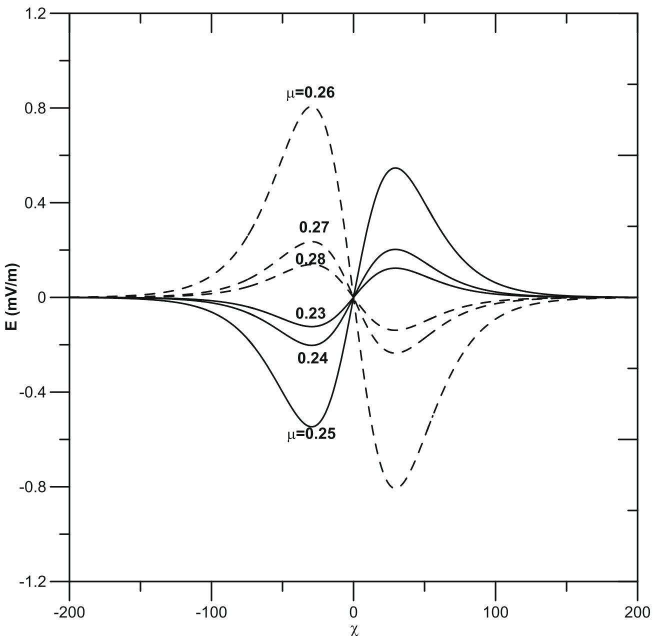

Fig. 2. Variation of electric field of the EASWs E with χ for different low temperature ion density, μ. Here $T_l = 10\ \text{eV}, n_0 = 0.1\ \text{cm}^{-3}$ and other parameters are same as used in Fig. 1.

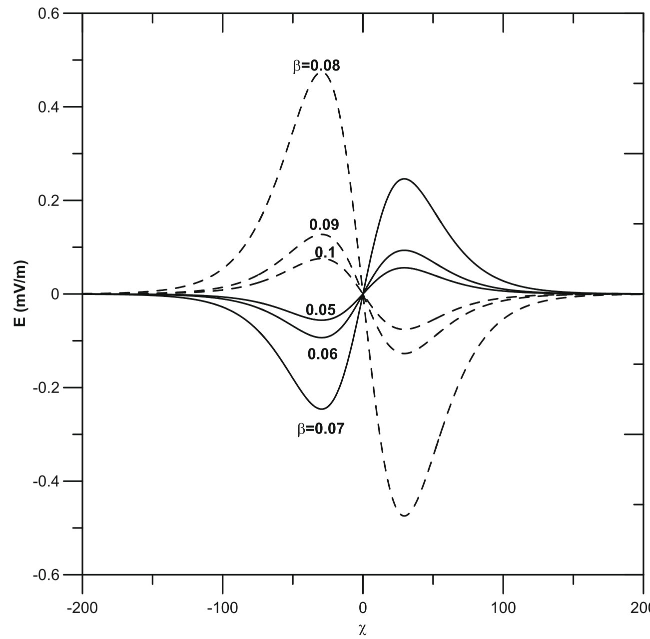

Fig. 4. Variation of electric field of the EASWs E with χ for different ion temperature ratio, β. Here $T_l = 10\ \text{eV}, n_0 = 0.1\ \text{cm}^{-3}$ and other parameters are same as used in Fig. 3.

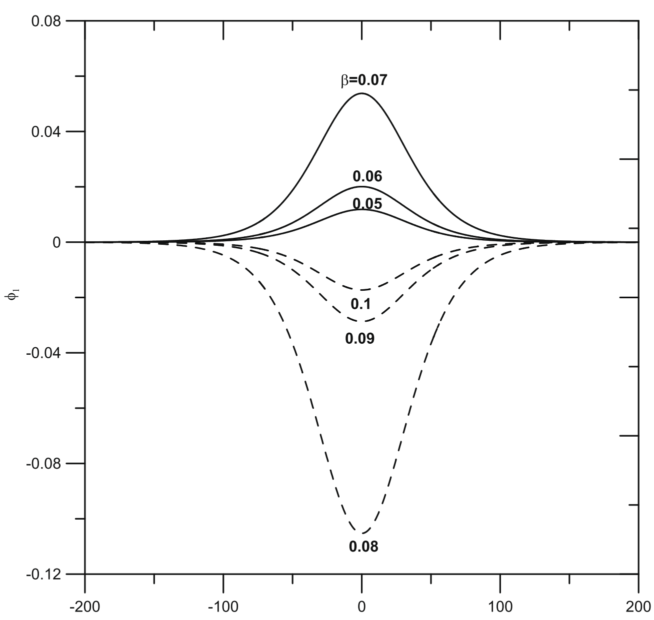

Fig. 3. Variation of potential ϕ_1 with χ for different ion temperature ratio, β. Other parameters are: $\mu = 0.2$, $\delta M = 0.001$.

tial changes into rarefactive EASWs. In case of compressive EASWs, it is found that the width and the amplitude of the mode increase with β, whereas in case of rarefactive EASWs the width and amplitude both decrease with an increase of β. In this case the bipolar electric field structures associated with the compressive and rarefactive potentials are shown in Fig. 4, as a function of χ for different ion temperature ratio, β.

5. Discussion

We have applied our theory to illustrate the features of EASWs in plasma sheet boundary layer of the Earth's magnetotail. In particular, this region is known for the presence of multi plasma species, e.g. cold electrons, multi temperature ions/ion beams. This is, therefore, one of the situation where the present model may be applied. The earthward and anti-earthward streaming of ions of density $\leqslant 0.1\ \text{cm}^{-3}$ with temperatures 0.1–1 keV have been observed within plasma sheet boundary layer (Lui et al., 1983; De Coster and Frank, 1979; Williams, 1981). Sharp et al. (1981, 1982) observed cold ion streams of densities $\leqslant 0.1\ \text{cm}^{-3}$ and temperature of 50 eV or less in the plasma sheet region. The possible role of both these ions in the generation of electrostatic noise in the plasma sheet boundary is discussed by Dusenbery and Lyons, 1985. In our calculation of electric field associated with the electrostatic structures as shown in Fig. 1, we consider the temperature of cold and hot ions as $T_l = 10$ eV and $T_h \simeq 200$ eV. In this case, the electric field associated with the compressive and rarefactive EASWs is found to be in the range of (0.01–0.5) mV/m and (0.01–0.8) mV/m, respectively.

In this paper we have discussed the generation of small amplitude EASWs in a three component unmagnetized plasma (cold plasma electrons, low temperature ions and high temperature ions). The properties of these small amplitude electron acoustic waves are studied by employing the reductive perturbation technique. Recently Kakad et al. (2007) discussed a mechanism for the nonlinear evolution of arbitrary amplitude EASWs in a 4-component plasma system (cold electron, cold beam electron and two different temperature ion species). For certain plasma conditions they have discussed the possibility of co-existence of rarefactive and compressive modes in the plasma sheet boundary layer. In their model the EASWs were driven by the cold electron beams. However, the present model deals with the evolution of EASWs in the absence of cold electron beams. For the interpretation of the compressive EASWs, the previous models considered either presence of electron beams (Berthomier et al., 2000; El-Taibany, 2005) or hot electron inertia (Verheest et al., 2005). However, the present study show that the presence of cold electron beams and the hot electron inertia is not a necessary condition for the generation of compressive EASWs structures if there are two temperature (low and high) ions along with cold plasma electrons in a plasma system.

The estimated electric field of the electrostatic strictures studied here are in good agreement with the observed solitary wave structures in the Earth's plasma sheet boundary layer. Therefore, this model could be useful to interpret the compressive and rarefactive bipolar pulses of the Broadband Electrostatic Noise (BEN) type emissions in the regions where the electron beams are not present.

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