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BROADBAND ELECTROSTATIC NOISE DUE TO NONLINEAR ELECTRON-ACOUSTIC WAVES

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ABSTRACT

Nonlinear propagation of electron-acoustic waves is examined in an unmagnetized, four-component plasma consisting of hot Maxwellian electrons, fluid cold and beam electrons and ions. Solitary structures which are a possible final stage of the electron-acoustic wave growth are obtained. The soliton amplitude and width are numerically obtained. The results are compared with the spiky structures of the broadband electrostatic noise observed in the auroral region of the Earth's magnetosphere. The model predicts parallel electric fields $\sim$ (10-400) mV/m with typical half widths of the structures $\sim$ a few Debye lengths.

INTRODUCTION

Satellite measurements in the auroral and other regions of the magnetosphere have shown bursts of broadband electrostatic noise (BEN) emissions with frequencies upto and higher than the electron plasma and cyclotron frequencies. High time resolution data analysis have indicated that BEN consists of small scale, large amplitude, magnetic field aligned electric fields in the auroral acceleration region (Temerin et al., 1982, Bostrom et al., 1988, Mozer et al., 1997, Ergun et al., 1998, Bounds et al., 1999, Pottelette et al., 1999), in the plasma sheet boundary layer (PSBL) (Matsumoto, 1994), in the Earth's high altitude polar magnetosphere (Franz et al., 1998), in polar cap boundary layer (PCBL) (Tsurutani et al., 1998), and on cusp field lines (Catell et al., 1999). These large amplitude spiky structures in the parallel electric field have been interpreted in terms of either solitons (Temerin et al., 1982) or phase space holes (Omura et al.,1994). The observations have shown two different types of solitary potential structures, i.e., ion and electron solitary potential structures propagating parallel to the magnetic field. These ion and electron solitary potential structures propagate at $\sim$ 10-300 km/s and $\sim$ 500-5000 km/s, respectively. The parallel scale sizes of these structures are in the range of 100-1000 m and the electric field amplitude can be of as high as a few hundreds mV/m.

A detailed study of the properties of the broadband electrostatic noise in the dayside auroral zone has been presented by Dubouloz et al., (1991a). The presence of high frequency broadband electrostatic noise and large amplitude of electric field suggest that nonlinear effects play an important role in the generation of BEN. Dubouloz et al., (1991b, 1993) studied the electron-acoustic solitons (EAS) with two electron-component (cold and hot) and motionless ions and showed that the high frequency broadband electrostatic noise in dayside auroral zone can be generated by the EAS. The effect of upward propagating electron beams which are commonly observed in the dayside auroral zone was not taken into consideration. Berthomier et al., (1998) studied the characteristics of ion acoustic solitary waves and weak double layers in a two-electron temperature auroral plasma. Recently, electron-acoustic solitons (Berthomier et al., 2000) have been studied in an electron-beam plasma system and a parametric study of the small amplitude solitons has been presented.

We extend the model of Dubouloz et al., (1991b) by including the electron beam dynamics, ion motion, adiabatic equation of state and study the arbitrary amplitude electron-acoustic solitons in an unmagnetized auroral plasma. The effect of various parameters such as soliton velocity, electron beam drift velocity, hot electron density etc. on the soliton amplitude is studied. In the next section, theoretical analysis of the model is presented and results are discussed and concluded in the following sections.

THEORETICAL ANALYSIS

We consider a homogeneous, unmagnetized plasma with four components, namely, nondrifting Maxwellian hot electrons and fluid cold electrons, an electron beam drifting along the x-axis, the direction of propagation of the electron-acoustic waves, and fluid ions. The governing normalized equations for the model are given by

$$n_h = n_{oh} exp(\phi), \tag{1}$$

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j v_j) = 0, \tag{2}$$

$$\frac{\partial v_j}{\partial t} + v_j \frac{\partial v_j}{\partial x} + \frac{1}{\mu_j n_j}\frac{\partial P_j}{\partial x} - \frac{Z_j}{\mu_j}\frac{\partial \phi}{\partial x} = 0, \tag{3}$$

$$\frac{\partial P_j}{\partial t} + v_j \frac{\partial P_j}{\partial x} + 3P_j \frac{\partial v_j}{\partial x} = 0, \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_h + n_c + n_b - n_i, \tag{5}$$

where the subscript h denotes the hot electrons and $j = c, b, i$ represents cold and beam electrons and ions respectively, $Z_j = \pm 1$ for electrons and ions respectively, and $\mu_j = m_j/m_e$. Normalizations are as follows; densities with respect to the total unperturbed electron density $n_o = n_{oc} + n_{oh} + n_{ob} = n_{oi}$, velocities with hot electron thermal velocity $v_{th} = \sqrt{T_h/m_e}$, lengths with $\lambda_d = \sqrt{T_h/4\pi n_o e^2}$, time with $\sqrt{4\pi n_o e^2/m_e}$, thermal pressure with $n_o T_h$ and potential ϕ with T_h/e.

A linear dispersion relation for electron-acoustic waves can be obtained by linearising and solving a set of Eqs. (1)-(5) and is given by

$$\omega^2 = \frac{k^2(n_{oc}/n_{oh})v_{th}^2}{1 + k^2\lambda_{dh}^2}\left[1 + 3k^2\lambda_{dc}^2 + \frac{n_{ob}}{n_{oc}}(1 + 3k^2\lambda_{db}^2)\right], \tag{6}$$

whereλ_{dh},λ_{dc} and λ_{db} are the Debye lengths for hot, cold and beam elections, respectively. In deriving the linear dispersion relation (6) for the electron-acoustic waves, we have for simplicity assumed beam drift velocity $v_o = 0$ and taken $(n_{0h}T_b/n_{oc}T_h) << 1$. It is interesting to note that in the absence of electron beam, our linear dispersion relation (6) reduces to Eq. (1) of Berthomier et al., (2000).

We seek solutions to Eqs. (1)-(5) that are stationary in a frame moving with velocity V. Using the transformation $\xi = x - Vt$ in Eqs. (1)-(5) and applying the appropriate boundary conditions, Poisson equation (5) can be integrated to yield energy integral,

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0, \tag{7}$$

where $V(\phi)$ is the Sagdeev potential given by

$$
\begin{aligned}
V(\phi) &= n_{oh}\left(1-e^{\phi}\right)+n_{oc}\left[V^2-\frac{V}{\sqrt{2}}\left\{V^2+3T_c+2\phi+\sqrt{(V^2+3T_c+2\phi)^2-12T_cV^2}\right\}^{1/2}\right] \\
&+ n_{oc}T_c\left[1-2\sqrt{2}V^3\left\{V^2+3T_c+2\phi+\sqrt{(V^2+3T_c+2\phi)^2-12T_cV^2}\right\}^{-3/2}\right] \\
&+ n_{ob}\Bigg[(V-v_o)^2-\frac{(V-v_o)}{\sqrt{2}}\Bigg\{(V-v_o)^2+3T_b+2\phi \\
&+ \sqrt{\left\{(V-v_o)^2+3T_b+2\phi\right\}^2-12T_b(V-v_o)^2}\Bigg\}^{1/2}\Bigg] \\
&+ n_{ob}T_b\Bigg[1-2\sqrt{2}(V-v_o)^3\Bigg\{(V-v_o)^2+3T_b+2\phi \\
&+ \sqrt{\left\{(V-v_o)^2+3T_b+2\phi\right\}^2-12T_b(V-v_o)^2}\Bigg\}^{-3/2}\Bigg] \\
&+ \mu_i\left[V^2-\frac{V}{\sqrt{2}}\left\{V^2+\frac{3T_i}{\mu_i}-\frac{2\phi}{\mu_i}+\sqrt{\left(V^2+\frac{3T_i}{\mu_i}-\frac{2\phi}{\mu_i}\right)^2-\frac{12T_iV^2}{\mu_i}}\right\}^{1/2}\right] \\
&+ T_i\left[1-2\sqrt{2}V^3\left\{V^2+\frac{3T_i}{\mu_i}-\frac{2\phi}{\mu_i}+\sqrt{\left(V^2+\frac{3T_i}{\mu_i}-\frac{2\phi}{\mu_i}\right)^2-\frac{12T_iV^2}{\mu_i}}\right\}^{-3/2}\right] \qquad (8)
\end{aligned}
$$

where v_o is the drift velocity of the electron beam and $\mu_i = m_i/m_e$. The soliton solutions of Eq. (7) exist when the usual conditions $V(\phi)=0, dV(\phi)/d\phi = 0$ at $\phi = 0$ and $V(\phi) < 0$ for $0 < |\phi| < |\phi_o|$ are satisfied, where ϕ_o is the maximum amplitude of the solitons. We numerically solve Eq. (7) for the soliton amplitude ϕ and Sagdeev potential $V(\phi)$. The results are presented in the next section.

We now apply our results to the Viking satellite observations in the auroral region of the Earth's magnetosphere using the parameters from Dubouloz et al. (1993) for the burst b (refer to Table 2 of Dubouloz et al., 1993). The parameters are as follows: the densities of cold, hot and beam electrons respectively are $n_{oc} = 0.2$ cm^{-3}, $n_{oh} = 1.5$ cm^{-3}, and $n_{ob} = 1.0$ cm^{-3}, normalized temperatures are $T_c/T_h = 0.001 = T_i/T_h$, $T_b/T_h = 0.01$ and normalized electron beam drift velocity $v_o/v_{th} = 0.1$. Figure 1 shows the variation of Sagdeev potential $V(\phi)$ with the potential ϕ and for the above mentioned parameters for various values of the normalized soliton velocity V/v_{th} indicated on the curves. The numerical results show that soliton solutions exist for $1 \le V/v_{th} \le 1.5$ for the above mentioned parameters. The results are not affected significantly when T_c/T_h is increased to 0.01.

Figure 2 shows the variation of the soliton amplitude ϕ with ξ for different values of the soliton velocity V/v_{th}. The other parameters are same as for Figure 1. It is obvious from Figure 2 that soliton width (maximum amplitude) decreases (increases) when the soliton velocity increases which can also be seen from Figure 1.

Figure 3 shows the variation of soliton amplitude with ξ for the parameters of Figure 1 for different values of normalized electron beam drift velocity v_o/v_{th} for $V/v_{th} = 1.4$. The soliton amplitude decreases with the increase in the drift velocity and the soliton solutions do not exist for $v_o/v_{th} \ge 0.5$. Also, small values of T_b/T_h are required in order to obtain soliton solutions (not shown).

Figure 4 shows the variation of soliton amplitude for different values of hot electron density for $V/v_{th} = 1.4$. The other parameters are the same as for Figure 1. The soliton solutions are obtained for 0.75 cm^{-3} $\le n_{oh} \le 1.65$cm^{-3}. Outside this range soliton solutions do not exist. The soliton amplitude increases with the increase in the hot electron density.

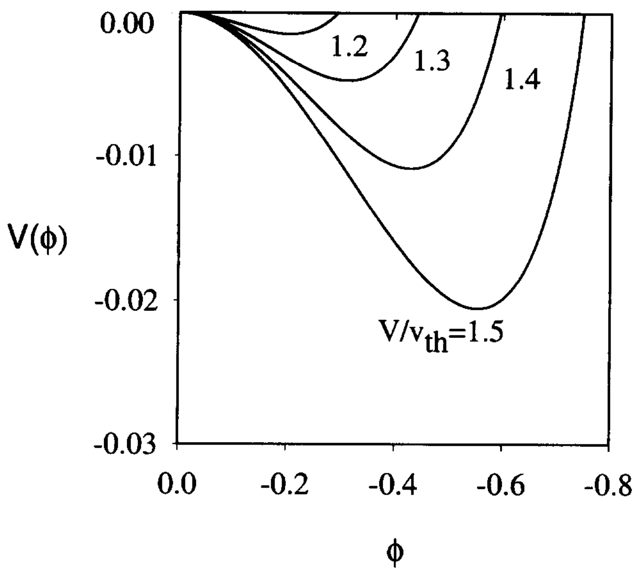

Fig.1. Sagdeev potential $V(\phi)$ vs potential ϕ for $n_{oc} = 0.2\ cm^{-3}$, $n_{oh} = 1.5\ cm^{-3}$, $n_{ob} = 1.0\ cm^{-3}$, $T_c/T_h = 0.001 = T_i/T_h$, $T_b/T_h = 0.01$, and $v_o/v_{th} = 0.1$ for various values of normalized soliton velocity V/v_{th} as indicated on the curves.

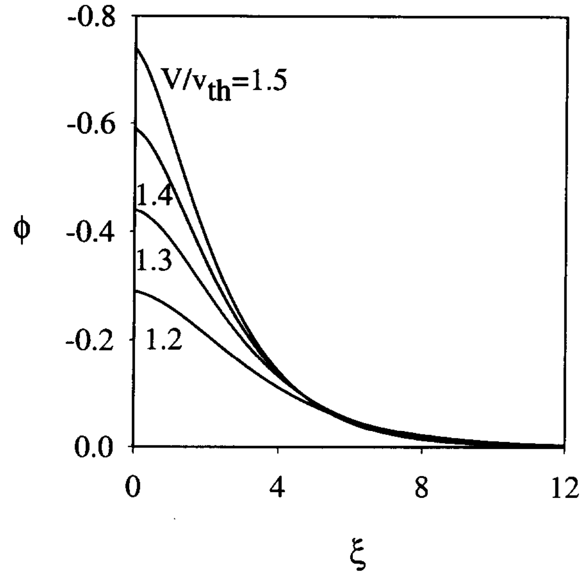

Fig. 2. Potential ϕ vs ξ for various values of normalized soliton velocity V/v_{th} for the parameters of Fig. 1.

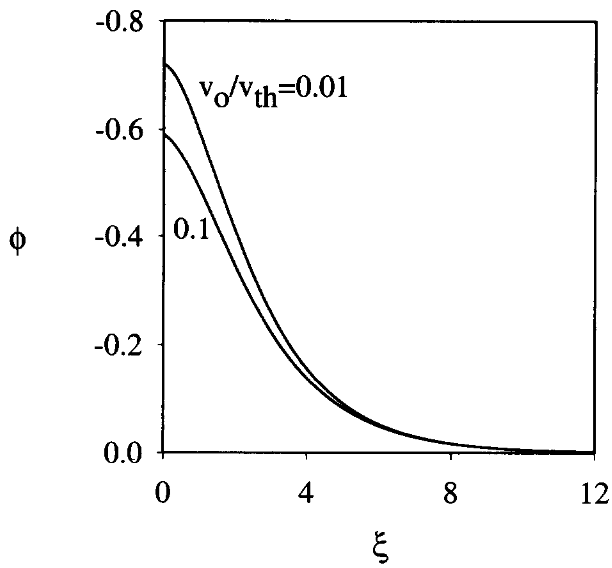

Fig. 3. Potential ϕ vs ξ for various v_o/v_{th} values for $V/v_{th} = 1.4$. Other parameters are same as of Fig. 1.

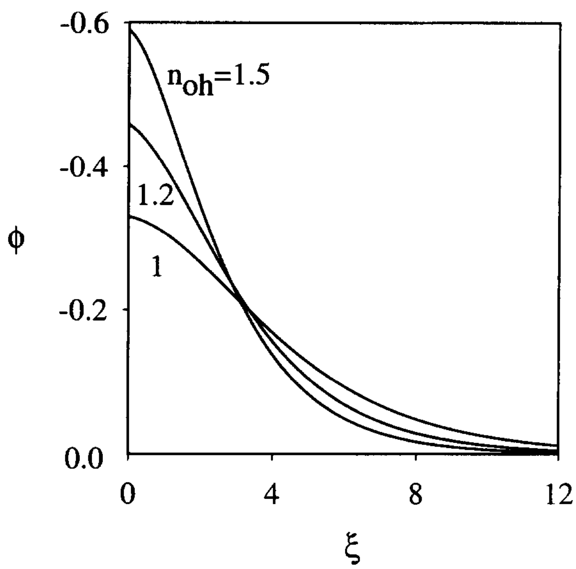

Fig. 4. Potential ϕ vs ξ for various values of n_{oh} (cm^{-3}) for $V/v_{th} = 1.4$. Other parameters are same as of Fig. 1.

Further, for three-component plasma without the electron beam one requires hot electron density to be of the order of cold electron density to get the soliton solutions (not shown). On the otherhand, for the case without the cold electron component the, soliton solutions exist for wide range of beam electron density 1.1 $\text{cm}^{-3} \le n_{ob} \le 2.3$ cm^{-3}. However, soliton amplitude decreases with the increase in the electron beam density (not shown).

For the typical parameters, namely, $n_{oc} = 0.2$ cm^{-3}, $n_{oh} = 1.5$ cm^{-3}, $n_{ob} = 1.0$ cm^{-3}, $T_h = 100eV$, $T_b \sim 1eV$, $T_c = T_i \sim 1eV$ and $v_o/v_{th} = 0.01 - 0.5$, i.e., corresponding to the electron beam drift velocity $v_o \sim 40 - 2100$ km/s, the typical electric field associated with solitons are found to be $E_{||} \sim$ 10-400 mV/m and typical soliton half widths of $\sim$ 1-4 Debye length. The typical velocities of the electron-acoustic solitary structures are obtained as $\sim$ (1.1-1.5) v_{th}, i.e., $\sim$ (4500-6300)km/s. One should however, check these predictions with the observed beam parameters in the auroral acceleration region.

CONCLUSIONS

We have studied the nonlinear propagation of electron-acoustic waves in an unmagnetized auroral plasma consisting of hot Maxwellian electrons, drifting beam electrons and nondrifting cold electrons and ions. It is emphasized that amplitude of the electron-acoustic solitons as well as parametric regime where the solitons can exist is sensitive to the beam parameters. The increase of the beam velocity can lead to the reduction of the electron-acoustic soliton amplitude. The increase in beam to hot electron temperature ratio, T_b/T_h, narrows down the range of Mach numbers for which soliton exists and also reduces the amplitude of the solitons. Further, the number density of the hot electrons, n_{oh}, controls the properties of the electron-acoustic solitons. The soliton amplitude increases with the increase of n_{oh} upto a certain value, beyond which electron-acoustic soliton ceases to exist. It would be worthwhile to re-examine the data from Viking and other spacecraft to test the predictions of our model.

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