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Obliquely propagating ion-acoustic solitons and supersolitons in four-component auroral plasmas

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Abstract

Arbitrary amplitude nonlinear low frequency electrostatic soliton and supersoliton structures are studied in magnetized four-component auroral plasmas composed of a cold singly charged oxygen-ion fluid, Boltzmann distribution of hot protons and two distinct group of electron species. Using the Sagdeev pseudo-potential technique, the characteristics of obliquely propagating nonlinear structures are investigated analytically and numerically. The model supports the evolution of soliton and supersoliton structures in the auroral acceleration region. Depending on the parametric region, the positive and negative potential solitons coexists at lower Mach numbers, but at higher Mach numbers only negative potential solitons and supersolitons can exist. The presence of hot protons restricted the Mach number of the nonlinear structures to exist only at the subsonic region. The present investigation concurs with the Swedish Viking satellite observations in the auroral region.

Keywords: Low frequency waves; Solitons and supersolitons; Magnetized plasmas; Mach numbers; Auroral zone; Earth's magnetosphere

1. Introduction

S3-3 satellite reported the observations of electrostatic solitary structures having bipolar electric field parallel to the magnetic field in the auroral acceleration region (Temerin et al., 1982). Since these first observations of nonlinear structures (Temerin et al., 1982), several spacecraft missions have reported similar observations in different regions of the Earth's magnetosphere, e.g. in the auroral acceleration region (Boström et al., 1988; Dubouloz et al., 1991; Mozer et al., 1997; Ergun et al., 1998; Bounds et al., 1999), plasma sheet boundary layer (PSBL) (Matsumoto et al., 1994; Omura et al., 1994, 1999), the bow shock (Bale et al., 1998), magnetopause and on cusp field lines (Franz et al., 1998; Cattel et al., 1999), Polar cap boundary layer (Tsurutani et al., 1998), auroral kilometric radiation (AKR) source region (Pottelette et al., 1999), magnetosheath (Pickett et al., 2003, 2005) and the Earth's foreshock region (Shin et al., 2008).

Recent FAST satellite measurements in the auroral acceleration region have revealed that the auroral plasmas are characterised by several ion species (Strangeway et al., 1998; McFadden et al., 1999, 2003; Crymley et al., 2001). Such solitary structures have also been studied in multi-ion laboratory plasmas (Chakrabarti et al., 2002; Sauer et al., 2002).

Theoretical studies of these nonlinear solitary waves have received much attention (Bharuthram and Shukla, 1985, 1986; Mann et al., 1997; Ghosh and Lakhina, 2004;

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Lakhina et al., 2011; Rufai et al., 2012, 2014a, 2015). Raychaudhuri et al. (1985) presented a model to describe the propagation of ion-acoustic solitons in a multi-ion component plasma with two electron temperatures. Their studies were based on a comparison of kinetic theory and reductive perturbation technique applied to the warm ion fluid equations to obtain a Korteweg deVries (KdV) equation, in order to explain the frequent appearance of heavy ions components in the solar wind and in the vicinity of comets. Using the fluid dynamics approach, Jain et al. (1990) investigated the effect of second-ion species on the characteristics of large-amplitude ion-acoustic double layers in a collisionless, unmagnetized multispecies plasma consisting of two distinct groups of Maxwellian distributed electrons and two cold-ion fluids. Ghosh and Lakhina (2004) presented studies in four-component magnetized and unmagnetized plasma models consisting of heavy and light mass ions species, governed by the fluid dynamic equations with two temperature electron populations. Using the Sagdeev pseudo-potential technique, they described the POLAR satellite observations of large amplitude solitary waves in the auroral plasmas. Recently, plasma models consisting of multi-ions species have been used by many authors to study the nonlinear ion- and electron-acoustic waves observed in the plasma sheet boundary layer of the Earth's magnetosphere (Ghosh and Lakhina, 2006; Kakad et al., 2007; Hellberg and Verheest, 2008; Djebli and Marif, 2009). Moreover, Lakhina et al. (2008) studied large amplitude nonlinear ion- and electron-acoustic waves in an unmagnetized four-component plasma system consisting of cold electrons, hot beam electrons, cold ions and hot beam ions. They treated all the species as mobile by using the fluid dynamic equations for each species and employed the Sagdeev potential technique to obtain the energy integral equation. Sabry (2009) investigated the effect of positron concentration and temperature ratios on the characteristics of the large and small amplitude ion-acoustic solitary waves and double layers in multicomponent plasma consisting of hot positrons, cold fluid ions, and two-electron temperature distributions. Verheest et al. (2012) used the Korteweg–de Vries (KdV) equation to establish the evolution equations for nonlinear electrostatic structures, in a plasma composed of two cold (positively and negatively charged) ion species and Boltzmann electrons.

A new class of electrostatic solitary waves known as nonlinear super-solitons have been very recently reported by several authors, for example Dubinov and Kolotkov (2012), Verheest et al. (2013), Maharaj et al. (2013), Rufai et al. (2014b) and Singh and Lakhina (2015). In this case, a series of soliton solutions have an upper bound represented by a double layer, which is immediately coupled to a (super) soliton. In a recent paper, Verheest et al. (2013) have conjectured that the associated electric field signatures of supersolitons should be observable in available or future space and laboratory observations in three-component plasmas (e.g. Cluster spacecraft measurements Pickett et al., 2004 shows a supersoliton electric field wedged between two regular bipolar structures). Also, in the laboratory experiments, these should appear as "distorted" bipolar electric field forms, which may be resolved, e.g. via proton imaging (Romagnani et al., 2008), among other techniques. Recently, Rufai et al. (2014b) obtained nonlinear electrostatic supersoliton structures in magnetized three-component plasmas. In multi-ion component, Rufai (2015) showed the existence of ion-acoustic soliton and supersoliton structures in a magnetized nonthermal plasma consisting of cold ions fluid, Boltzmann hot ions and nonthermal hot electrons.

In the present study, we investigate low frequency electrostatic soliton and supersoliton structures in a magnetized plasma consisting of a cold singly charged oxygen-ion fluid and Boltzmann distributions of hot protons and two-electron species of different temperatures. The present investigation is an extension to our earlier work (Rufai et al., 2012) by including a hot proton in the analysis. In Section 2, the theoretical model is presented and the Sagdeev pseudo-potential equation for the nonlinear structures is derived. The characteristics of the nonlinear structures is shown in Section 3. The numerical results are presented in Section 4. Then, conclusions are drawn in the last section.

2. Theoretical model

We consider a four-component, homogeneous, collisionless, and magnetized plasma comprising of Boltzmann distributions of hot protons (N_{hi}, T_{hi}), cool electrons (N_{ce}, T_{ce}) and hot electrons (N_{he}, T_{he}) and a cold singly charged oxygen-ion fluid (N_{ci}). Wave propagation is in the z direction at an angle θ to the external magnetic field $\mathbf{B_0}$, which is assumed to be in the (x, z)-plane. Then, the normalized set of the governing equations are

$$n_{ce} = f\exp(\alpha_{ce}\psi), \tag{1}$$

$$n_{he} = (1-f)\exp(\alpha_{he}\psi), \tag{2}$$

$$n_{hi} = (1-g)\exp(-\alpha_{hi}\psi), \tag{3}$$

$$\frac{\partial n_{ci}}{\partial t} + \frac{\partial(n_{ci}v_x)}{\partial x} + \frac{\partial(n_{ci}v_z)}{\partial z} = 0, \tag{4}$$

$$\frac{\partial v_x}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_x = -\frac{\partial\psi}{\partial x} + v_y, \tag{5}$$

$$\frac{\partial v_y}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_y = -v_x, \tag{6}$$

$$\frac{\partial v_z}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_z = -\frac{\partial\psi}{\partial z}. \tag{7}$$

For the four component plasma, the quasineutrality condition at equilibrium is given by $N_{ci0} + N_{hi0} = N_{ce0} + N_{he0} = N_0$. We have used the following normalizations in the above set of equations: density is normalized by the total density n_0 , velocity normalized by the effective ion acoustic speed $c_s = (T_{eff}/m_i)^{1/2}$, electrostatic potential,

ϕ is normalized by T_{eff}/e, distance by the effective ion Larmor radius, $\rho_i = c_s/\Omega$, time t normalized by the inverse of the oxygen-ion gyro-frequency Ω^{-1}, $(\Omega = eB_0/m_i c)$. We also define the temperature ratio $\tau = T_{ce}/T_{he}$, cool electron density ratio $f = N_{ce0}/N_o$, the ion density ratio, $N_{ci0}/N_0 = g, T_{eff} = T_{ce}/(f + (1-f)\tau)$ is the effective electron temperature, $\alpha_{ce} = T_{eff}/T_{ce}, \alpha_{he} = T_{eff}/T_{he}, \alpha_{hi} = T_{eff}/T_{hi}$.

Since our study focuses on low frequency phenomena, we assume the charge neutrality condition, with

$$n_{ci} = n_{ce} + n_{he} - n_{hi} = fe^{\alpha_{ce}\psi} + (1-f)e^{\alpha_{he}\psi} - (1-g)e^{-\alpha_{hi}\psi}. \quad (8)$$

We transform all the dependent variables in Eq. (4)–(7) to a single independent variable in a moving frame, i.e. $\xi = (\alpha x + \gamma z - Mt)/M$, where $M = V/c_s$ is the Mach number, $\alpha = \sin\theta, \gamma = \cos\theta; \theta = \theta(z, \mathbf{B_0})$. Integrate Eq. (1)–(8) with appropriate boundary conditions for solitary wave structures (namely, $n_{ci} \to g, \psi \to 0$, and $d\psi/d\xi \to 0$ at $\xi \to \pm\infty$). Then, we obtain a single dimensionless nonlinear differential equation in terms of the oxygen-ion density n_{ci} and electrostatic potential ψ as,

$$\frac{d}{d\xi}\left(\frac{d\chi(\psi)}{d\xi}\right) = \frac{M^2}{g}(n_{ci} - g) - \frac{\gamma^2}{g^2} n_{ci} H(\psi) \quad (9)$$

where

$$\chi(\psi) = \left(\psi + \frac{M^2}{2n_{ci}^2}\right) \quad (10)$$

and

$$\frac{d\chi(\psi)}{d\xi} = \left(1 - \frac{M^2}{n_{ci}^3} g^2 N'_{ci}(\psi)\right)\frac{d\psi}{d\xi} \quad (11)$$

with n_{ci} given by Eq. (8) and

$$H(\psi) = \frac{f}{\alpha_{ce}}(e^{\alpha_{ce}\psi} - 1) + \frac{1-f}{\alpha_{he}}(e^{\alpha_{he}\psi} - 1) + \frac{1-g}{\alpha_{hi}}(e^{-\alpha_{hi}\psi} - 1), \quad (12)$$

$$N'_{ci}(\psi) = \alpha_{ce} f e^{\alpha_{ce}\psi} + \alpha_{he}(1-f)e^{\alpha_{he}\psi} + \alpha_{hi}(1-g)e^{-\alpha_{hi}\psi}. \quad (13)$$

Multiplying both sides of Eq. (9) by $2d\chi(\psi)/d\xi$ and integrating once with appropriate boundary conditions, we obtain

$$\frac{1}{2}\left(\frac{d\chi(\psi)}{d\xi}\right)^2 + V(\psi, M) = 0. \quad (14)$$

Eq. (9) can be expressed in terms of ψ and M using Eq. (10), as

$$\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2 + V(\psi, M) = 0 \quad (15)$$

where the Sagdeev pseudo-potential $V(\psi, M)$ is given by

$$V(\psi, M) = -\left[1 - \frac{M^2 g^2 N'_{ci}(\psi)}{n_{ci}^3}\right]^{-2} \times \left[-\frac{M^4(n_{ci} - g)^2}{2n_{ci}^2} - M^2(1-\gamma^2)\psi + \frac{M^2 H(\psi)}{g} - \frac{\gamma^2 H^2(\psi)}{2g^2} - \frac{\gamma^2 M^2 H(\psi)}{n_{ci}}\right]. \quad (16)$$

Eq. (15) is the "energy balance equation" for a pseudo-particle of unit mass in a conservative force field, with velocity $d\psi/d\xi$ at position ψ, time ξ in a pseudo-potential $V(\psi, M)$.

3. Nonlinear characteristics

We examine the Sagdeev potential $V(\psi, M)$ in Eq. (16) to determine the behaviour and necessary conditions for the existence of the nonlinear low frequency potential structures. In doing so we use the soliton boundary conditions (Rufai, 2015) i.e. $V(\psi, M) = 0$, $dV(\psi, M)/d(\psi) = 0$, $d^2V(\psi, M)/d(\psi)^2 < 0$ at $\psi = 0; V(\psi, M) = 0$ at $\psi = \psi_m$, and $V(\psi, M) < 0$ for ψ lying between 0 and ψ_m. If $\psi_m < \psi < 0$, rarefactive solitary (RS) waves exist and it $0 < \psi < \psi_m$, compressive solitary (CS) waves exist, where ψ_m is the maximum amplitude of the solitons. For the formation of double layer structure, one more additional condition must be satisfied, i.e, $\left.\frac{dV(\psi,M)}{d\psi}\right|_{\psi=\psi_m} = 0$. Then, the supersoliton exists when there is an accessible root of the Sagdeev potential beyond the double layer.

The soliton condition $d^2V(\psi, M)/d\psi^2 < 0$ at $\psi = 0$ can be written as

$$\left.\frac{d^2V(\psi, M)}{d\psi^2}\right|_{\psi=0} = \frac{M^2 - M_0^2}{M^2 - M_1^2} < 0 \quad (17)$$

where

$$M_0^2 = \frac{g\gamma^2}{f\alpha_{ce} + (1-f)\alpha_{he} + (1-g)\alpha_{hi}} \quad (18)$$

is the critical Mach number and the upper bound of the Mach number is

$$M_1^2 = \frac{g}{f\alpha_{ce} + (1-f)\alpha_{he} + (1-g)\alpha_{hi}}. \quad (19)$$

For $\gamma = \cos\theta \neq 0 : \gamma^2 = \cos^2\theta < 1$; which implies $M_0 < M_1$, then if $M > M_1 \Rightarrow M > M_0$ from which $M^2 - M_0^2 > 0$ and $M^2 - M_1^2 > 0$, consequently (17) is not satisfied.

Similarly, if $M < M_0 \Rightarrow M < M_1$ from which $M^2 - M_0^2 < 0$ and $M^2 - M_1^2 < 0$, once again (17) is not satisfied. Thus, the solutions existence for low frequency electrostatic wave, only if the condition (17) is satisfied

$$M_0 < M < M_1 \quad (20)$$

Since, $f\alpha_{ce} + (1-f)\alpha_{he} = 1$ and $g < 1$, Eq. (20) becomes

$$\gamma\sqrt{\frac{g}{1 + (1-g)\alpha_{hi}}} < M < \sqrt{\frac{g}{1 + (1-g)\alpha_{hi}}}, \quad (21)$$

which determines the existing domain of the corresponding nonlinear structures. It is interesting to note that the nonlinear solutions can exists only in the subsonic Mach number region with $M_1 < 1$ as the upperbound for the existence region of Mach numbers (cf. Eq. (21)). This is consistent with the findings of Ghosh and Lakhina (2004) for the four component magnetized plasma model consisting of two fluid dynamic equations for ion species (i.e, hydrogen ions and oxygen beam ions) and two temperature electrons. They reported the existence of the nonlinear solutions in the subsonic Mach number regime. We note that in the absence of the hot proton species, i.e, $g = 1$, the model reduce to Rufai et al. (2012).

4. Numerical results

We now apply our results to the Viking satellite observations in the auroral region of the Earth's magnetosphere as reported by Berthomier et al. (1998). The parameters are as follows: the cool electron density and temperature $n_{ce} = 0.2\ \mathrm{cm}^{-3}$ and $T_{ce} = 1\ \mathrm{eV}$, hot electron density and temperature $n_{he} = 1.8\ \mathrm{cm}^{-3}$ and $T_{he} = 26\ \mathrm{eV}$, which gives the effective temperature $T_{eff} \approx 7\ \mathrm{eV}$ with a measured electric field amplitude less than 100 mV/m. We further present the numerical computations of the energy integral equation in (15) and the Sagdeev potential $V(\psi, M)$ given in (15) for standard plasma parameters such as Mach number M, cool electron density ratio f, cool to hot electron temperature ratio τ, propagating angle θ, oxygen ion relative density g and the hot proton temperature $\alpha_{hi} = T_{eff}/T_{hi}$.

Table 1 shows the unnormalized values of the soliton velocity V, electric field E, soliton width W, and pulse duration $\tau^* = W/V$ for various values of g and the Mach number range M, respectively. It is interesting to note that from Table 1 that compressive soliton (CS) structures can only exist for $g < 0.6$ for $\theta = 35^\circ$. Then the rarefactive soliton (RS) structures exist for $g > 0.6$. The rarefactive supersoliton (RSS) structures can be obtained for the range $0.7 \leqslant g \leqslant 0.9$. As g increasing, the Mach number (both the minimum M_0 and maximum M_1), the soliton velocity, width and the electric field tend to increase with g, but pulse duration decrease on each potential side. At $g = 1.0$ corresponding to the three-component plasma model.

The curves in Fig. 1 show the variation of the Sagdeev potential $V(\psi, M)$ against the normalized electrostatic potential ψ for different values of Mach number M. The fixed parameters are cool electron density, $f = 0.1$, cool to hot electron temperature, $\tau = 0.04$, cold oxygen ion density, $g = 0.6$ and propagating angle, $\theta = 15^\circ$. The coexistence of positive and negative soliton solutions is obtained and $M = 0.758$ is the upper limit of Mach numbers for positive solitons. The soliton amplitude ψ, increases with increasing Mach number M for both positive and negative potential solitons. It is interesting to pointed out that at $M = 0.741524$, a positive potential soliton coexists with a negative potential double layer and a supersoliton. The negative potential supersoliton is identified by a deep second well, beyond the double layer, in the Sagdeev potential. The curve $M = 0.745$ is a soliton. Further investigation shows that for this fixed parameters, only compressive soliton solutions can be obtained for the value of $g < 0.6$. Then for $g > 0.6$ the soliton solutions moved completely to exist at negative potential (rarefactive solitons). Fig. 2 shows the normalized electrostatic potential ψ against ξ for the parameters of Fig. 1. For $M = 0.735$, 0.738 and 0.74, the positive and negative ion-acoustic solitons coexist. The hodographs are plotted in Fig. 3, the curves show the variation of $d\psi/d\xi$ against ψ for the same parameters of Fig. 1. The curves

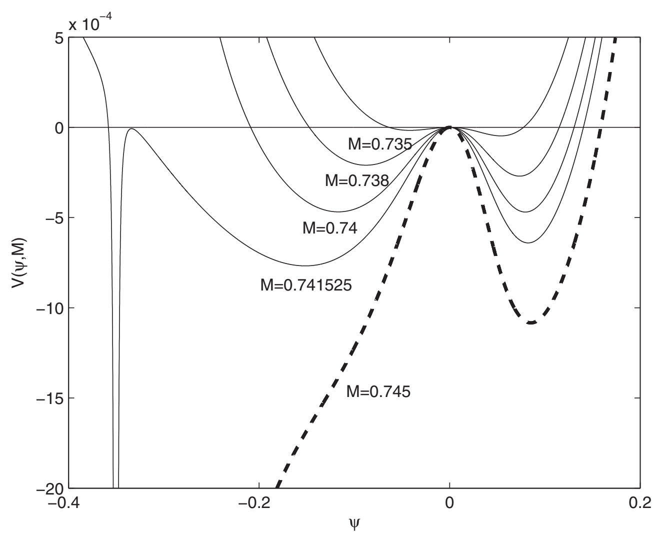

Fig. 1. Sagdeev potential, $V(\psi, M)$ vs normalized electrostatic potential ψ, for $\tau = 0.04$, $f = 0.1$, $g = 0.6$ and $\theta = 15^\circ$.

Table 1
Properties of ion-acoustic solitons, such as Soliton Velocity (V), Mach number range ($M_0 < M < M_1$), Electric Field (E), Soliton Width (W) and Pulse Duration (τ^*), for various values of cold ion density (g) with $\theta = 35^\circ$, normalized cool electron density $f = 0.1$, cool to hot electron temperature ratio $\tau = 0.04$, hot proton temperature $\alpha_{hi} = 0.1$ and Oxygen ion mass $m_i = 26.7616 \times 10^{-24}$ g.

g	$M_0 - M_1$	V (km s^{-1})	E (mV m^{-1})	W (m)	τ^* (ms)
0.5	0.566–0.690 (CS)	3.68–4.48	0.16–118.23	315.90–19.76	85.84–4.41
0.6	0.623–0.750 (CS)	4.05–4.87	2.57–156.15	193.70–17.81	47.83–3.66
0.7	0.676–0.71779 (RSS)	4.39–4.66	0.13–34.06	357.50–170.30	81.43–36.54
0.8	0.729–0.813055 (RSS)	4.74–5.28	0.77–43.93	169.00–197.44	35.65–31.71
0.9	0.779–0.913236 (RSS)	5.06–5.94	1.08–60.81	137.80–154.70	27.23–26.04
1.0	0.821–0.999 (RS)	5.34–6.49	0.16–94.14	244.66–56.55	45.82–8.71

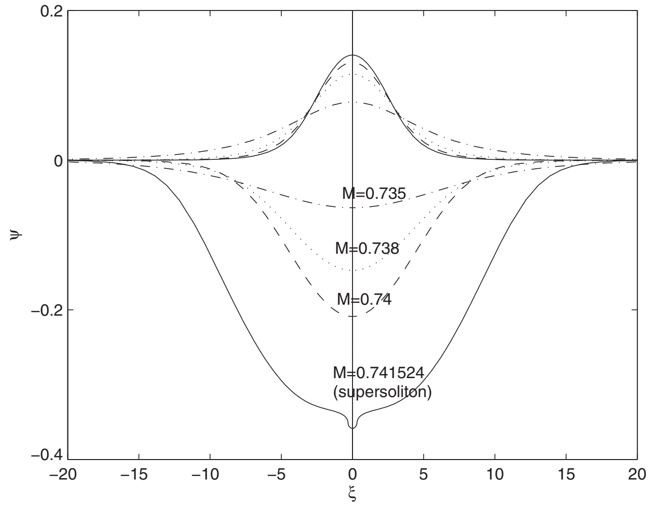

Fig. 2. Normalized electrostatic potential ψ vs ξ, for the same parameters of Fig. 1.

$M = 0.735, 0.738$, and 0.74 correspond to the conventional soliton, while $M = 0.741524$ is a supersoliton having much wider amplitude than conventional solitons. The curves also show that supersoliton have more inflection points than conventional soliton. The difference between the standard soliton and the supersoliton structures are clearly shown.

Fig. 4 shows the variation of the Sagdeev potential $V(\psi, M)$ versus ψ for different values of the cool electron number density f. The other fixed parameters are $\tau = 0.04$, $g = 0.9$, $\theta = 15^\circ$, and $M = 0.94$. Only negative potential solitons occur for the parameters considered here. It is observed that as the cool electron density increases (f increasing), the soliton amplitude increases. Numerical computations for the chosen set of parameters show that the soliton solutions are not possible beyond $f > 0.30$. This is consistent with Fig. 2(b) of Baboolal et al. (1990), where they showed that negative potential nonlinear structures can exist for cold electron density ratio f roughly between 0.02 and 0.35.

Fig. 5 shows the variation of the Sagdeev potential $V(\psi, M)$ versus the normalized electrostatic potential ψ for different values of angle of propagation θ. For the parameters considered here, only negative potential solitons and supersolitons occur. The chosen parameters are, $f = 0.1$, $g = 0.9$, $\tau = 0.04$ and $M = 0.94$. The curves show that as the angle of propagation θ increases, the soliton amplitude increases. At a propagating angle $\theta = 32.5242^\circ$ a supersoliton structure appears. Further investigation reveals that no soliton/double layer/supersoliton solution can be obtained for the value of angle of propagation above $\theta > 80^\circ$. Whereas for the case of four component magnetized plasma studied by Ghosh and Lakhina

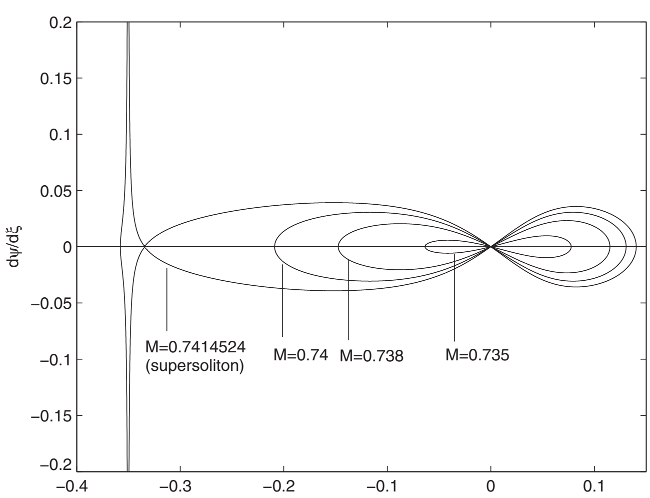

Fig. 3. The phase $d\psi/d\xi$ vs ψ, for the same parameters of Fig. 1.

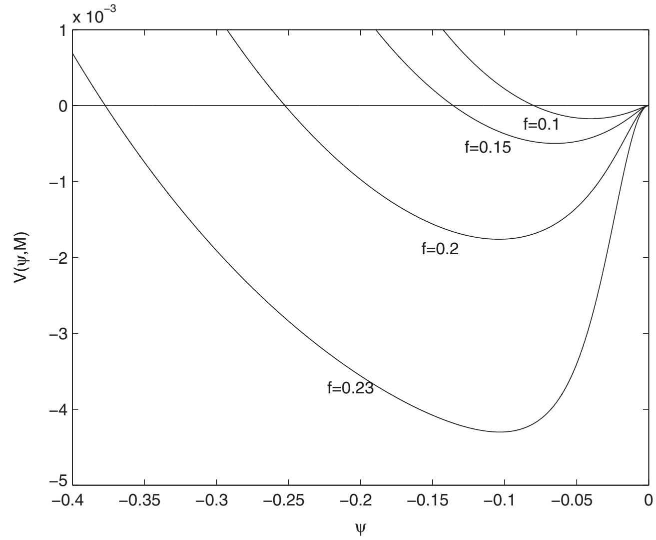

Fig. 4. Sagdeev potential, $V(\psi, M)$ vs normalized electrostatic potential ψ, for $\tau = 0.04$, $g = 0.9$, $\theta = 15^\circ$ and $M = 0.94$.

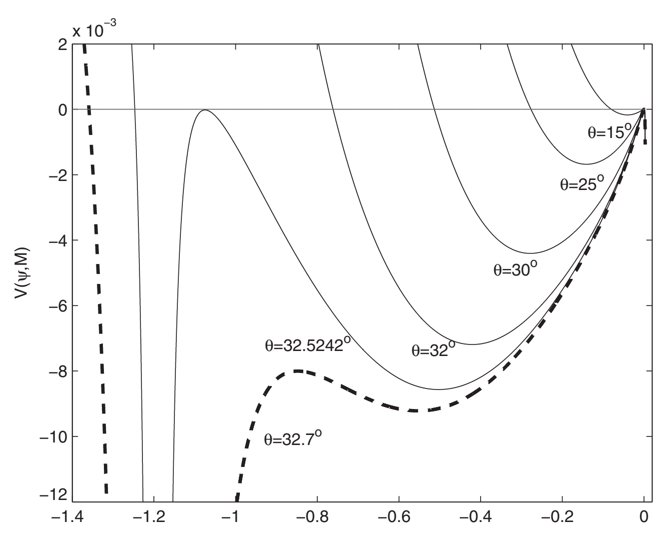

Fig. 5. Sagdeev potential, $V(\psi, M)$ vs normalized electrostatic potential ψ, for $\tau = 0.04$, $f = 0.1$, $g = 0.9$ and $M = 0.94$.

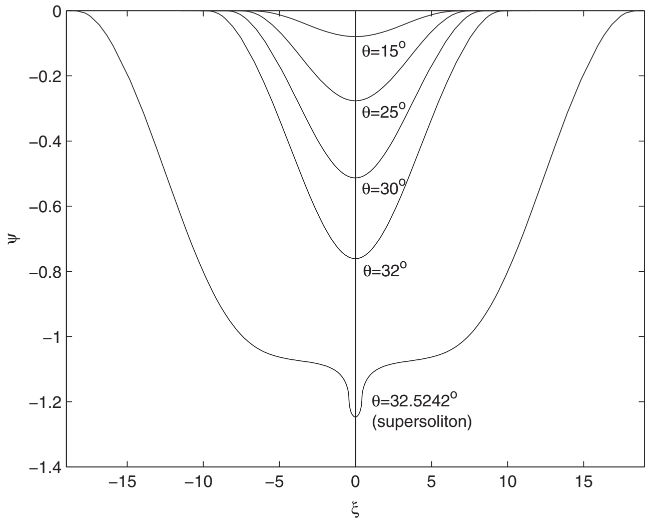

Fig. 6. Normalized electrostatic potential ψ vs ξ, for $\tau = 0.04$, $f = 0.1$, $g = 0.9$ and $M = 0.94$.

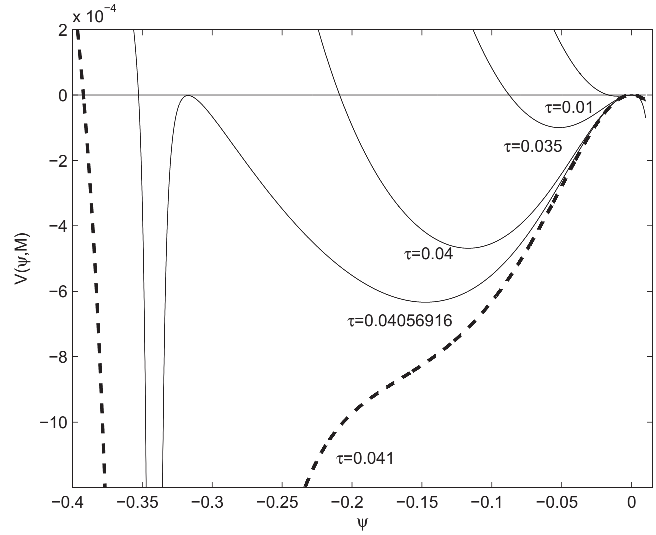

Fig. 8. Sagdeev potential, $V(\psi, M)$ vs normalized electrostatic potential ψ, for $f = 0.1$, $g = 0.6$, $\theta = 15^\circ$ and $M = 0.74$.

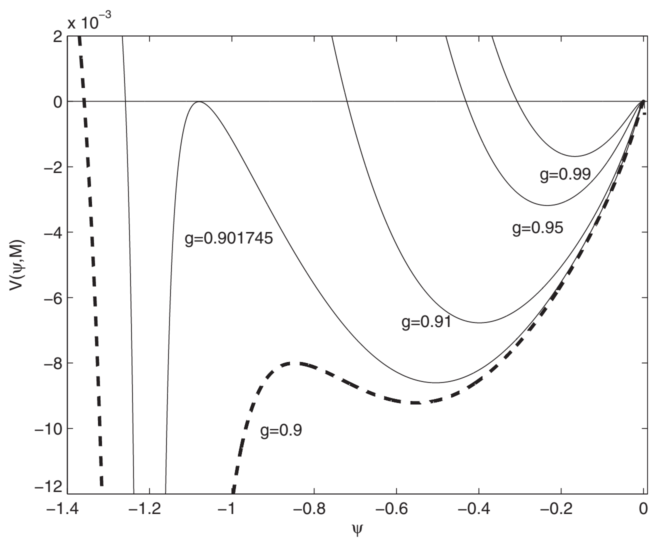

Fig. 7. Sagdeev potential, $V(\psi, M)$ vs normalized electrostatic potential ψ, for $\tau = 0.04$, $f = 0.1$, $M = 0.94$ and $\theta = 32.7^\circ$.

(2004), compressive and rarefactive solitary wave structures can only be found to exist for a wide range of θ. The corresponding electrostatic potential profiles of the solitons and supersoliton are plotted in Fig. 6 which clearly shows that the amplitude and width of the nonlinear structures are increasing with increase in θ.

As the oxygen ion relative density g increases, the wave amplitude of the negative potential supersolitons and solitons decreases, as shown in the Sagdeev potential $V(\psi, M)$ profiles presented in Fig. 7. It is interesting to point out that a supersoliton solution is accessible for fixed value of angle of propagation, $\theta = 32.7^\circ$ and other parameters of Fig. 5, at $g = 0.901745$. No soliton/double layer/supersoliton solution can exist below $g < 0.9$.

The variation of the Sagdeev potential $V(\psi, M)$ against ψ for different values of cool to hot electron temperature ratio $\tau = T_{ce}/T_{he}$ is shown in Fig. 8. For angle of propagation $\theta = 15^\circ$ and other fixed parameters are $f = 0.1$, $g = 0.6$, $M = 0.74$. The ion-acoustic wave amplitude increases with the increase in electron temperature ratio. The soliton solutions exist for cool to hot electron temperature ratio $\tau \geqslant 0.01$ and at $\tau = 0.04056916$ the supersoliton solution appears. The curve $\tau = 0.041$ is a supersoliton and no soliton/double layer/supersoliton solution can be found for $\tau > 0.041$.

5. Conclusion

The evolution of nonlinear ion-acoustic solitons and supersolitons in magnetized multi-component plasmas made up of cold singly charged oxygen-ion fluid, Maxwellian hot proton species and Boltzmann distributions of cool and hot electron populations are studied, using the Sagdeev pseudo-potential technique. This model is a modification of the three-component magnetized plasma studied by Rufai et al. (2012) by including a hot proton. The effects of the inclusion of the additional species in this plasma model were studied and the conditions under which the nonlinear structures can exist were found both analytically and numerically. The Mach number regime were found to exist only in the subsonic Mach number region. Whereas for the case of the three-component plasma studied in our early paper (Rufai et al., 2012), the nonlinear solutions have the maximum Mach number value of 1. For the four-component magnetized plasma model consisting of H^+ ions and O^+ beam ions governed by fluid equations and two Boltzmann electrons temperature. Ghosh and Lakhina (2004) showed the coexistence of both rarefactive and compressive solitons only. Numerical computation of our model revealed the coexistence of both compressive soliton and rarefactive supersoliton structures at lower values of the Mach number. But at the higher Mach number, only negative solitons and supersolitons are predicted.

Related to the spacecraft observations of nonlinear low frequency electrostatic structures in the auroral region of the Earth's magnetosphere, the measured parameters (Berthomier et al., 1998) were cool electron density and temperature $n_{ce} = 0.2\,\text{cm}^{-3}$ and $T_{ce} = 1\,\text{eV}$, hot electron density and temperature $n_{he} = 1.8\,\text{cm}^{-3}$ and $T_{he} = 26\,\text{eV}$, which gives the effective temperature $T_{eff} \approx 7\,\text{eV}$ with total electric field amplitude of less than 100 mV/m, width of about 100 m, about 20 ms of pulse duration and ≈ 10–50 km/s soliton velocities range respectively, see Table 1. For our typical computational parameters for supersoliton of Fig. 6, $g = 0.9$ $M = 0.94$ and $\theta = 32.5242^{\circ}$ the maximum electric field generated is about 58.1 mV/m and corresponding soliton width, pulse duration and speed comes out to be ~ 150 m, 24.6 ms and 6.1 km/s, respectively. The properties of the solitons/double layers/supersolitons predicted by our model are in good agreement with the properties of the solitary structures observed by Viking. Our findings may be helpful in improving the understanding of the nonlinear fluctuations in the auroral region of the Earth's magnetosphere.

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