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A New Method for Prediction of Peak Sunspot Number and Ascent Time of the Solar Cycle

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Abstract The purpose of the present study is to develop an empirical model based on precursors in the preceding solar cycle that can be used to forecast the peak sunspot number and ascent time of the next solar cycle. Statistical parameters are derived for each solar cycle using "Monthly" and "Monthly smoothed" (*SSN*) data of international sunspot number (R_{i}). Primarily the variability in monthly sunspot number during different phases of the solar cycle is considered along with other statistical parameters that are computed using solar cycle characteristics, like ascent time, peak sunspot number and the length of the solar cycle. Using these statistical parameters, two mathematical formulae are developed to compute the quantities $[Q_{\mathrm{C}}]_n$ and $[L]_n$ for each nth solar cycle. It is found that the peak sunspot number and ascent time of the $n+1$th solar cycle correlates well with the parameters $[Q_{\mathrm{C}}]_n$ and $[L]_n/[S_{\mathrm{Max}}]_{n+1}$ and gives a correlation coefficient of 0.97 and 0.92, respectively. Empirical relations are obtained using least square fitting, which relates $[S_{\mathrm{Max}}]_{n+1}$ with $[Q_{\mathrm{C}}]_n$ and $[T_{\mathrm{a}}]_{n+1}$ with $[L]_n/[S_{\mathrm{Max}}]_{n+1}$. These relations predict a peak of 74 ± 10 in monthly smoothed sunspot number and an ascent time of 4.9 ± 0.4 years for Solar Cycle 24, when November 2008 is considered as the start time for this cycle. Three different methods, which are commonly used to define solar cycle characteristics are used and mathematical relations developed for forecasting peak sunspot number and ascent time of the upcoming solar cycle, are examined separately.

Keywords Solar cycle prediction · Models · Sunspots

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1. Introduction

The prediction of solar cycle characteristics like the solar maximum amplitude, ascent time and duration of cycle is becoming important day by day as we are relying more on satellites. A number of models are available at present for the forecast of solar activity; however, the reliability of these models needs to be scrutinized before using them for the predictions. Generally, one needs to predict the peak sunspot number, the time of the occurrence of the peak sunspot number and the trend of the solar cycles with a better accuracy. Forecasting models are broadly divided into two groups: statistics-based and dynamo-based models. Statistical models are further divided in two categories which are based on *i*) extrapolation and *ii*) precursor method. In the precursor method the information on peak and ascent times for the upcoming solar cycle is available well in advance or at later time (Ohl, 1966; Feynman, 1982; Wilson 1990, 1992; Thompson, 1993; Obridko, 1995; Wilson, Hathaway, and Reichmann, 1998; Hathaway and Wilson, 2006; Kane, 2007). In the extrapolation method month by month prediction of the sunspot number or the shape of the solar cycle is obtained either by regression or by a curve fitting method, once the solar cycle is in progress (McNish and Lincoln, 1949; Hathaway, Wilson, and Reichmann, 1994; Kane, 1999). Different statistics-based techniques, which are currently used to forecast the solar activity, are examined by Hathaway, Wilson, and Reichmann (1999). Other models are based on the solar dynamo (Dikpati and Charbonneau, 1999; Dikpati, de Toma, and Gilman, 2006; Dikpati and Gilman, 2008). In a recent study the dynamo based model is combined with the precursor technique to provide predictions of future solar activity (Svalgaard, Cliver, and Kamide, 2005). Also in the study by Schatten (2005), the polar field precursor method is used which has a physical basis rather than just a numerical basis.

In the method proposed by Ohl (1966) it is observed that the minimum in the *aa* index correlates well with the maximum solar activity of the next solar cycle and yields a correlation coefficient of 0.90. However, these predictions are not available until the minimum is reached in the *aa* index, which may sometimes occur after the time of the minimum in the sunspot number. In the method given by Feynman (1982), geomagnetic activity index *aa* is separated into two components:

i) aa_{R} is proportional to sunspot number and
ii) aa_{I} is a "interplanetary component" attributed to changes in geomagnetic activity and solar wind.

It is found that aa_{I} gives a better correlation with the peak sunspot number. Also, aa_{I} reaches a maximum well before the time of the solar minimum. Thus the estimate for the peak sunspot number for future solar activity is available well in advance. Another method is Thompson's method (1993), which utilizes the number of geomagnetically disturbed days occurring in the previous solar cycle. In that study the author found that the number of disturbed days in the previous solar cycle correlates well with the sum of the peak sunspot number for the previous and the next solar cycles and it produces a correlation coefficient of 0.97. It should be noted that geomagnetic indices essentially indicate the extent of magnetic activity on Earth and crucially depend on the geo-effectiveness of the CMEs or solar flares and also on the interplanetary conditions (Dungey, 1961; Burton, McPherron, and Russell, 1975; Mayaud, 1980). Hence, geomagnetic activity indices do not simply reflect global solar activity. Secondly, the geomagnetic *aa* index is available since 1868, so the correlation coefficients obtained using the above-mentioned methods are based only on the data of Solar Cycles 12 – 23.

In the present work an empirical model is developed to predict the peak smoothed sunspot number and the ascent time for the following solar cycle by utilizing the information of the preceding solar cycle. The proposed method is based on the data of Solar Cycles 1 – 23. Data set and methodology used in the present work are described in Section 2. Results obtained using the proposed method are briefly described in Section 3 and the present work is discussed and concluded in Section 4.

2. Data Used and Methodology

International sunspot numbers (R_{i}) are produced by the Solar Influences Data Analysis Center (SIDC) for the sunspot index, at the Royal Observatory of Belgium. "Monthly" and "Monthly smoothed" (*SSN*) data of the international sunspot number are used in the present study, which are available from January 1749 and July 1749, respectively. These data can be downloaded from http://ngdc.noaa.gov or http://sidc.oma.be/sunspot-data. Each "Monthly smoothed" sunspot number is obtained by smoothing of monthly sunspot numbers with a 13-month filter centered on the month of interest with half weights for the months at the start and end (Hathaway, 2010). Sunspot numbers vary considerably from one month to another. Despite these variations, a periodic behavior is clearly observed in the sunspot numbers, which has a periodicity of about 7 – 11 yrs. The strength and characteristics of the solar cycle differ from one cycle to another. The end of the previous solar cycle is the start for the next solar cycle. Each solar cycle is characterized by the ascent time (T_{a}), descent time (T_{d}), length of the solar cycle (T_{cy}), solar minimum ($S_{\min}$) and solar maximum (S_{Max}). If $[ts]_n$ is the start time *i.e.* the time of occurrence of minimum $S_{\min}$, $[tp]_n$ is the time of occurrence of solar maximum and $[te]_n$ $(=[ts]_{n+1})$ is the end time for the nth solar cycle, then the characteristics of the nth solar cycle are defined thus: ascent time $[T_{\mathrm{a}}]_n = [tp]_n - [ts]_n$, descent time $[T_{\mathrm{d}}]_n = [ts]_{n+1} - [tp]_n$ and length of solar cycle $[T_{\mathrm{cy}}]_n = [T_{\mathrm{a}}]_n + [T_{\mathrm{d}}]_n$. It should be noted that an estimate of T_{a}, T_{d} and T_{cy} implicitly depends on how one defines the occurrence of maxima and minima in the solar cycle. In the present study, three different methods are used to determine the characteristics of the solar cycle.

In the first method *ts*, *tp*, $S_{\min}$, and S_{Max} for each solar cycle are estimated by considering mathematical minima and maxima in the "Monthly smoothed" sunspot number, which are downloaded from ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS/INTERNATIONAL/smoothed/SMOOTHED. It should be noted that if the same minimum (maximum) value occurs at more than one place then in such cases a minimum (maximum) occurring at the first place is considered as minimum (maximum) for that solar cycle. For example the same minimum value of 0 in the monthly smoothed sunspot number occurs from February 1810 to December 1810 for Solar Cycle 6. So, February 1810 is considered as the time of occurrence of the minima or the start time, *ts*, for Solar Cycle 6. Solar cycle characteristics estimated using the above method are compiled in Group I.

In Group II, we have taken the solar cycle characteristics from ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS/docs/maxmin.new, which are commonly used by the scientific community for solar cycle studies. In this method, the number of spotless days and the frequency of occurrence of "old" and "new" cycle spot groups are taken into account along with minima in the monthly smoothed sunspot number, while determining the minimum for a given solar cycle. Hence T_{a} and T_{d}, estimated by considering only mathematical minima and maxima in the monthly smoothed sunspot number (*i.e.* Group I), may vary from estimates of T_{a} and T_{d} obtained from the website (*i.e.* Group II).

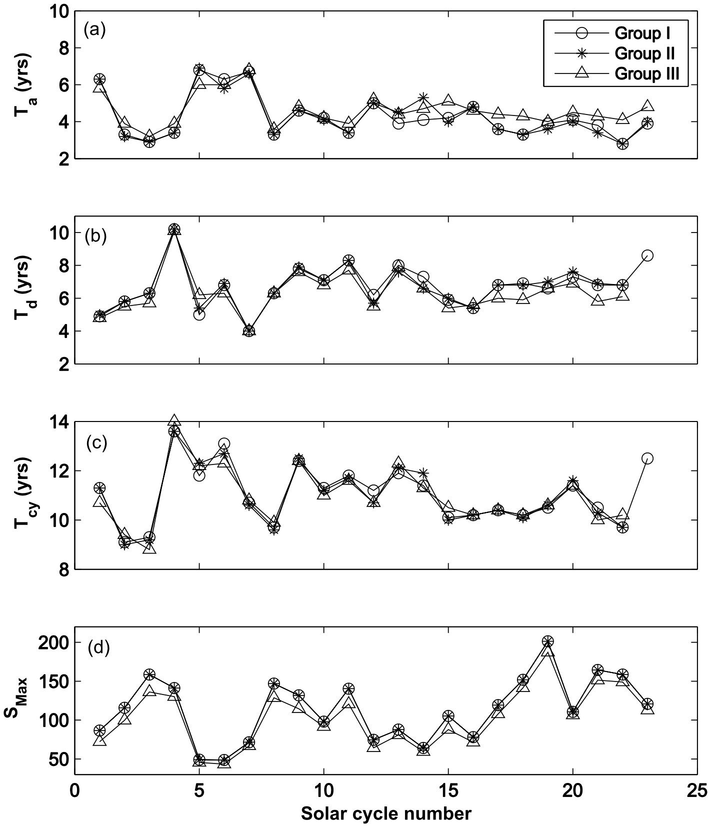

Figure 1 The length of the (a) ascent, (b) descent, (c) full cycle and the (d) peak sunspot number for Solar Cycles 1 – 23 are plotted for Groups I, II and III.

The third method for determining solar cycle characteristics is based on the smoothing of monthly sunspot numbers using Gaussian-shaped filter (Hathaway, Wilson, and Reichmann, 1999), which is given below with its relative weights:

$$W(t) = \mathrm{e}^{-0.3t^2/a^2} - \mathrm{e}^{-1.2}\left(2.2 - 0.3t^2/a^2\right) \quad (\text{Where } t = \pm 2a), \tag{1}$$

where a is the width of filter and t is the time from the center of the Gaussian filter. A Gaussian-shaped filter described in Equation (1) with a width of one year ($a = 12$ months) is used as a weight function and weighted averages of monthly sunspot number are obtained. In this method the center of the Gaussian filter coincides with a point where average is estimated. Weight and its first derivative both vanishes at $t = \pm 2a$. Thus one essentially needs 23 data points on either side of the center of Gaussian filter, if full-width at half maximum (FWHM) is 24. The solar cycle characteristics are determined by considering maxima and minima occurring at first place in the monthly sunspot numbers smoothed with Gaussian filter and compiled in Group III. Solar cycle characteristics obtained from mathematical treatment (Group I), website (Group II) and Gaussian filter smoothing (Group III) for Solar Cycles 1 – 23 are listed in Table 1. Ascent time, descent time, length of solar cycle and maximum sunspot number for Solar Cycles 1 – III are shown in Figure 1 for Groups I – III.

Table 1 Details of solar cycle characteristics: time of occurrence of minimum, *ts* (in year/month format), minimum sunspot number, S_{min}, ascent time, T_{a} (in years), length of solar cycle, T_{cy} (in years), peak sunspot number S_{Max} for Group I – III. T_{a} and T_{cy} of Group I and III mentioned below are rounded to first decimal place.

Cy.	Group I					Group II					Group III				
No.	*ts*	S_{min}	T_{a}	T_{cy}	S_{Max}	*ts*	S_{min}	T_{a}	T_{cy}	S_{Max}	*ts*	S_{min}	T_{a}	T_{cy}	S_{Max}
1	1755/02	8.4	6.3	11.3	86.5	1755/03	8.4	6.3	11.3	86.5	1755/06	10.7	5.8	10.7	72.3
2	1766/05	11.2	3.3	9.1	115.8	1766/07	11.2	3.2	9.0	115.8	1766/02	18.6	3.9	9.4	99.8
3	1775/06	7.2	2.9	9.3	158.5	1775/07	7.2	2.9	9.2	158.5	1775/07	14.4	3.2	8.8	136.2
4	1784/09	9.5	3.4	13.6	141.2	1784/09	9.5	3.4	13.6	141.2	1784/05	15.5	3.9	14.0	130.1
5	1798/04	3.2	6.8	11.8	49.2	1798/04	3.2	6.9	12.3	49.2	1798/05	5.5	6.0	12.2	45.5
6	1810/02	0.0	6.3	13.1	48.7	1810/08	0.0	5.8	12.7	48.7	1810/07	0.8	6.0	12.3	43.4
7	1823/03	0.1	6.7	10.7	71.5	1823/04	0.1	6.6	10.6	71.7	1822/11	3.4	6.8	10.8	66.9
8	1833/11	7.3	3.3	9.7	146.9	1833/11	7.3	3.3	9.6	146.9	1833/09	12.2	3.6	9.9	128.7
9	1843/07	10.6	4.6	12.4	131.9	1843/07	10.5	4.6	12.5	131.9	1843/08	15.6	4.8	12.4	114.6
10	1855/12	3.2	4.2	11.3	98.0	1856/01	3.2	4.1	11.2	97.9	1856/01	7.3	4.2	11.0	91.6
11	1867/03	5.2	3.4	11.8	140.3	1867/03	5.2	3.4	11.7	140.5	1867/01	12.9	3.9	11.6	120.7
12	1878/12	2.2	5.0	11.2	74.6	1878/11	2.2	5.0	10.7	74.6	1878/08	5.8	5.2	10.7	64.3
13	1890/02	5.0	3.9	11.9	87.9	1889/08	5.0	4.5	12.1	87.9	1889/04	6.4	4.4	12.3	81.0
14	1902/01	2.7	4.1	11.4	64.2	1901/09	2.6	5.3	11.9	64.2	1901/08	4.7	4.7	11.3	59.5
15	1913/06	1.5	4.2	10.1	105.4	1913/08	1.5	4.0	10.0	105.4	1912/11	3.0	5.1	10.5	87.8
16	1923/07	5.6	4.8	10.2	78.1	1923/08	5.6	4.8	10.2	78.1	1923/05	9.5	4.6	10.2	71.5
17	1933/09	3.5	3.6	10.4	119.2	1933/10	3.4	3.6	10.4	119.2	1933/07	7.7	4.4	10.4	107.9
18	1944/02	7.7	3.3	10.2	151.8	1944/03	7.7	3.3	10.1	151.8	1943/12	14.2	4.3	10.2	141.5
19	1954/04	3.4	3.9	10.5	201.3	1954/04	3.4	3.6	10.6	201.3	1954/02	11.6	4.0	10.6	187.2
20	1964/10	9.6	4.1	11.4	110.6	1964/11	9.6	4.0	11.6	110.6	1964/09	15.5	4.5	11.4	106.5
21	1976/03	12.2	3.8	10.5	164.5	1976/07	12.2	3.4	10.3	164.5	1976/02	16.1	4.3	10.0	151.4
22	1986/09	12.3	2.8	9.7	158.5	1986/10	12.3	2.8	9.7	158.5	1986/02	16.7	4.1	10.2	149.0
23	1996/05	8.0	3.9	12.5	120.8	1996/05	8.0	4.0	–	120.8	1996/04	12.7	4.8	–	112.8
24	2008/11	1.7	–	–	–	–	–	–	–	–	–	–	–	–	–

Table 2 Description of parameters derived from "Monthly smoothed" sunspot number (*SSN*) for ascending and descending phase of solar cycle. Time t is in years and $t_2 - t_1 > 0$.

Parameter	Start time for averaging	End time for averaging
A_1	$t_1 = ts$	$t_2 = ts + 0.5$
A_2	$t_1 = ts + T_a * 0.5 - 0.25$	$t_2 = ts + T_a * 0.5 + 0.25$
A_3	$t_1 = tp - 0.5$	$t_2 = tp$
D_1	$t_1 = te - 0.5$	$t_2 = te$
D_2	$t_1 = te - T_d * 0.5 - 0.25$	$t_2 = te - T_d * 0.5 + 0.25$
D_3	$t_1 = tp$	$t_2 = tp + 0.5$

It is seen that T_a, T_d, T_{cy}, and S_{Max} in Group II vary slightly as compared to Group I. However, the solar cycle characteristics in Group III vary considerably as compared to Group I. It should be noted that the information of descent time and the length of Solar Cycle 23 is available only for Group I as the mathematical minimum of 1.7 in the monthly smoothed sunspot number is approached in November 2008 (*i.e.* $[ts]_{24} = 2008.875$). Initially, the solar cycle characteristics from Group I are used with "Monthly" and "Monthly smoothed" data of the international sunspot number, and statistical parameters are computed during different phases of each solar cycle. Using these statistical parameters two formulae are developed to compute the quantities Q_C and L for each solar cycle. Parameters which are used to develop these formulae are described below.

In the first step, averages and standard deviations are estimated for different time durations for each solar cycle by utilizing the "Monthly" sunspot number data. $\overline{S}_{t_1,t_2}$ is the average of monthly sunspot number from starting time t_1 to end time t_2. $\overline{\sigma}_{t_1,t_2}$ is the standard deviation in the calculated mean $\overline{S}_{t_1,t_2}$. Secondly each solar cycle is divided into six parts of equal length, $T_{cy}/6$, and then the mean and standard deviations are calculated for each of these six parts, which are denoted by S_n and σ_{s_n} (where $n = 1 - 6$). S_n gives the average of the sunspot numbers falling between time $t_1 = ts + (n-1)T_{cy}/6$ and $t_2 = ts + (n)T_{cy}/6$. Apart from this, the "Monthly smoothed" sunspot number is used and three parameters A_1, A_2, A_3 and D_1, D_2, D_3 are derived from the ascending and descending phase of each solar cycle, respectively. These parameters are described in Table 2. A_1 and D_1 give an estimate of the sunspot number close to the start and end of the solar cycle. A_2 and D_2 provide the sunspot number near the middle of the solar cycle in the ascending and descending phase, respectively. A_3 and D_3 supply the information of the sunspot number just before and after the time of peak sunspot number. It should be noted that the average of the sunspot numbers between starting time t_1 and end time t_2 are computed such that $t_1 \le t < t_2$, where t is the time of the sunspot number.

The detailed parameters described above are computed for each solar cycle using solar cycle characteristics of Group I, and three formulae are developed to compute the quantities Q, Q_C and L. These formulae are given below:

$$Q = \frac{\overline{S}_{t_1=te-T_a,t_2=te}}{T_a} \times \frac{\overline{\sigma}_{t_1=tp,t_2=te}}{S_{Max}} \times \left(\frac{T_a T_d}{T_{cy}} + T_{cy}\right), \tag{2}$$

$$Q_C = Q \times \left[\frac{\sigma_{s_2}\sigma_{t_1=tp,t_2=tp+T_d*0.5}}{S_2(D_3 - D_2)} + \frac{\sigma_{s_5}\sigma_{t_1=ts+T_a*0.5,t_2=tp}}{S_5(A_3 - A_2)} + \frac{(\sigma_{s_2} + \sigma_{s_5})}{S_{Max}} + \frac{S_6}{S_4} + \frac{\sigma_{t_1=ts,t_2=ts+T_d*0.5} - \sigma_{t_1=ts,t_2=ts+T_a*0.5}}{S_{Max}}\right], \tag{3}$$

$$L = \left[\frac{T_{cy} * \sigma_{t_1=ts,t_2=tp}}{\overline{S}_{t_1=ts,t_2=tp}}\right]. \tag{4}$$

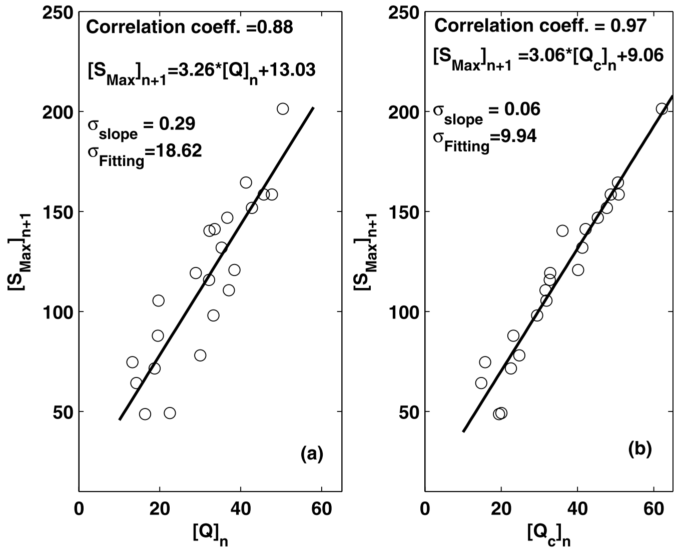

Figure 2 Plot of peak sunspot number for $n+1$th solar cycle, $[S_{\text{Max}}]_{n+1}$ as a function of quantities (a) $[Q]_n$ and (b) $[Q_{\text{C}}]_n$ estimated using solar cycle characteristics of Group I for each nth solar cycle, where $n = 1-22$ is the solar cycle number.

The quantities Q, Q_{C} have the dimension of sunspot number and are used to forecast the peak sunspot number of the next solar cycle. On the other hand, L has the dimension of time and is used to forecast the ascent time of the next solar cycle, provided that the forecast of the peak sunspot number for that solar cycle is known.

3. Results

3.1. Forecasting the Peak Sunspot Number of the Solar Cycle

The dependence of the peak sunspot number of the next solar cycle on the parameters Q and Q_{C} of the previous solar cycle is examined for Group I. A plot of the peak sunspot number for Solar Cycles 2 – 23 as a function of Q and Q_{C} for Solar Cycles 1 – 22 is shown in Figure 2. $[Q]_n$, $[Q_{\text{C}}]_n$ indicate computed quantities Q and Q_{C} for the nth solar cycle and $[S_{\text{Max}}]_{n+1}$ gives the peak sunspot number for the $n+1$th solar cycle. It is observed that the peak sunspot number for Cycles 2 – 23 correlates well with the estimated quantity Q for Solar Cycles 1 – 22 and the correlation coefficient of 0.88 is obtained. However, it is seen that when $[S_{\text{Max}}]_{n+1}$ is plotted against $[Q_{\text{C}}]_n$, the correlation improves significantly and a correlation coefficient of 0.97 is obtained. A least square fitting of $[Q]_n$, and $[Q_{\text{C}}]_n$ with $[S_{\text{Max}}]_{n+1}$ yields the following relations:

$$[S_{\text{Max}}]_{n+1} = 3.26 \times [Q]_n + 13.03, \tag{5}$$

$$[S_{\text{Max}}]_{n+1} = 3.06 \times [Q_{\text{C}}]_n + 9.06. \tag{6}$$

For Solar Cycle 24 a mathematical minimum of 1.7 is approached in November 2008, which gives an estimate of 20.76 and 21.14 for $[Q]_{23}$ and $[Q_{\text{C}}]_{23}$, respectively. Use of Equation (5)

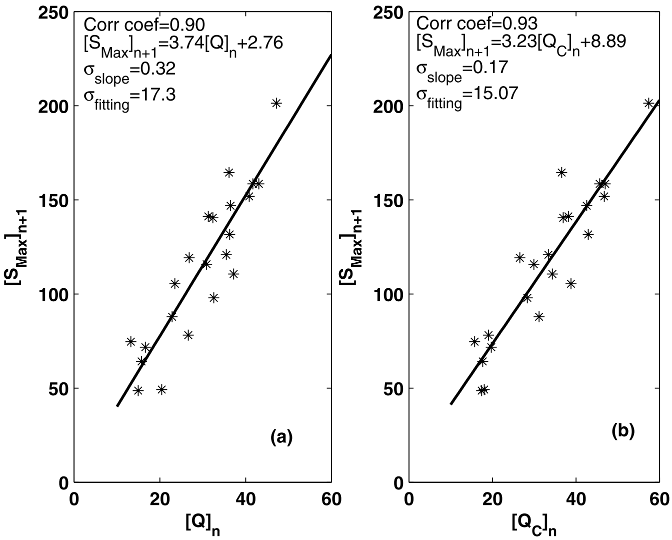

Figure 3 Plot of peak sunspot number for $n + 1$th solar cycle, $[S_{\mathrm{Max}}]_{n+1}$ as a function of quantities (a) $[Q]_n$ and (b) $[Q_{\mathrm{C}}]_n$ estimated using solar cycle characteristics of Group II for each nth solar cycle, where $n = 1-22$ is the solar cycle number.

and (6) yields a peak sunspot number of 81 ± 19 and 74 ± 10, respectively, for Solar Cycle 24. As Q_{C} correlates better with S_{Max} as compared to Q, the prediction of the peak sunspot number based on Q_{C} is considered for Solar Cycle 24. Here the error in the least square fitting is considered as the error in the predicted value.

Solar cycle characteristics are utilized to calculate the averages and standard deviations during different phases of the solar cycle. Hence the solar cycle characteristics play an important role in the process of computation of the parameters Q and Q_{C}. It is mentioned above that three different methods are used to get the solar cycle characteristics, which are compiled in Groups I – III. Here the dependence of the peak sunspot number on the quantities Q and Q_{C} is investigated for Group II and III as well. Solar cycle parameters of Groups II and III are used separately and, through the same mathematical treatment, Q and Q_{C} are estimated for Solar Cycles 1 – 22. The peak sunspot number for Solar Cycles 2 – 23 is plotted as a function of estimated Q and Q_{C} for Solar Cycles 1 – 22, which is shown in Figures 3 and 4, for Group II and III, respectively. It is found that the peak sunspot number for $n + 1$th solar cycle is in good correlation with Q and Q_{C} of the nth solar cycle and gives a correlation coefficient of 0.90 and 0.93 (0.85 and 0.88), respectively, for Group II (III). The correlation coefficient of $[S_{\mathrm{Max}}]_{n+1}$ with $[Q]_n$ increases to 0.90, whereas with $[Q_{\mathrm{C}}]_n$ it decreases to 0.93 for Group II as compared to Group I. For Group III the correlation coefficient of S_{Max} with Q and Q_{C} both decreases to 0.85 and 0.88 as compared to Group I. Nevertheless, the correlation coefficient between $[S_{\mathrm{Max}}]_{n+1}$ and $[Q_{\mathrm{C}}]_n$ is found to be greater than 0.90 for Groups I and II. The least square fitting of $[S_{\mathrm{Max}}]_{n+1}$ with $[Q]_n$ and $[Q_{\mathrm{C}}]_n$ gives the following relations for Groups II and III.

$$\text{Group II:}\quad [S_{\mathrm{Max}}]_{n+1} = 3.74 \times [Q]_n + 2.76, \tag{7}$$

$$[S_{\mathrm{Max}}]_{n+1} = 3.23 \times [Q_{\mathrm{C}}]_n + 8.89, \tag{8}$$

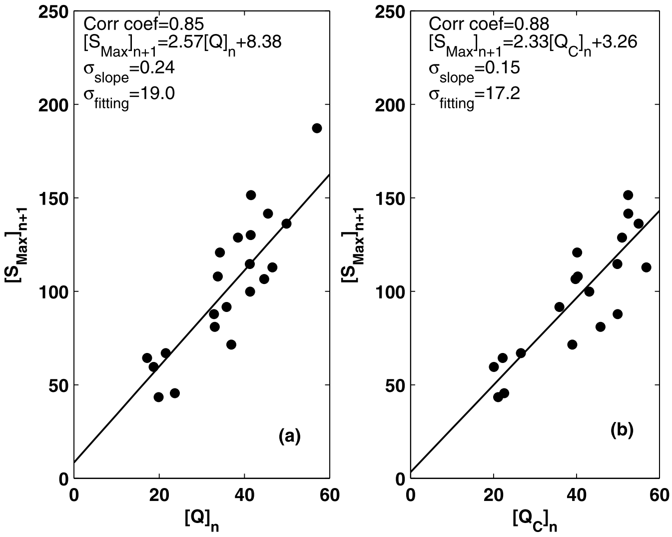

Figure 4 Peak sunspot number of $n+1$th solar cycle, $[S_{\mathrm{Max}}]_{n+1}$ as a function of quantities (a) $[Q]_n$ and (b) $[Q_{\mathrm{C}}]_n$ estimated using solar cycle characteristics of Group III for each nth solar cycle, where $n = 1-22$ is the solar cycle number.

$$\text{Group III:} \quad [S_{\mathrm{Max}}]_{n+1} = 2.57 \times [Q]_n + 8.38, \tag{9}$$

$$[S_{\mathrm{Max}}]_{n+1} = 2.33 \times [Q_{\mathrm{C}}]_n + 3.26. \tag{10}$$

The estimates of $[Q]_{23}$ and $[Q_{\mathrm{C}}]_{23}$ for Group II are not available, as T_{cy} for Solar Cycle 23 is not available on the website. Similarly $[Q]_{23}$ and $[Q_{\mathrm{C}}]_{23}$ are not estimated for Group III, as the monthly sunspot number smoothed with a 24 month FWHM Gaussian filter has not yet reached the minimum for Solar Cycle 24. The mathematical formulae given by Equations (2) – (3) are based on the statistical parameters which are estimated using solar cycle characteristics like ts, tp, T_{a}, T_{d}, S_{Max}, T_{cy}. As the estimate of these solar cycle characteristics varies from one group to another, it is expected that the correlation coefficients between $[Q_{\mathrm{C}}]_n$ and $[S_{\mathrm{Max}}]_{n+1}$ for Groups I – III will differ from each other. However, the correlation coefficient is found to be greater than 0.90 for Groups I and II. The highest correlation of 0.97 is obtained for Group I.

3.2. Forecasting the Ascent Time of Solar Cycle

In this investigation the ascent time and inverse of the peak sunspot number of the same solar cycle is compared for Groups I, II and III. Figure 5 illustrates the ascent time as a function of the inverse of the peak sunspot number of the same solar cycle for Groups I, II and III. It is noticed that $[T_{\mathrm{a}}]_n$ correlates positively with the inverse of $[S_{\mathrm{Max}}]_n$, which is basically the "Waldmeier effect". It states that the peak of the solar cycle is inversely proportional to the ascent time of the solar cycle (Waldmeier, 1935). The correlation coefficients of 0.80, 0.86, 0.77 are obtained for Group I, II and III, respectively. However, it is found that the ascent time T_{a} for the $n+1$th solar cycle gives a better correlation with the inverse of the peak sunspot number for the $n+1$th solar cycle when it is combined with the estimated parameter L for the nth solar cycle. Figure 6 illustrates a plot of the ascent time, $[T_{\mathrm{a}}]_{n+1}$ as

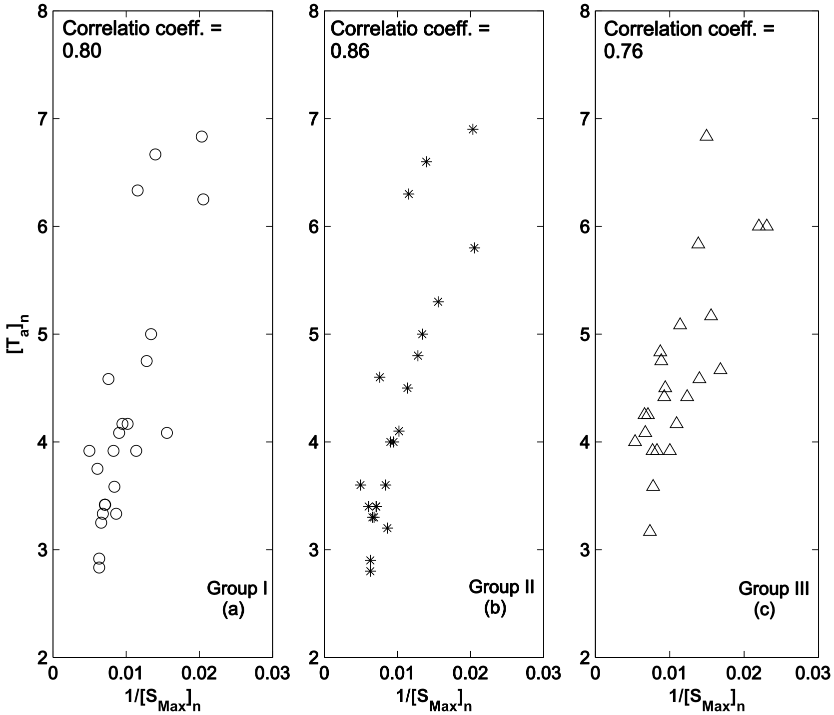

Figure 5 Ascent time of nth solar cycle, $[T_{\mathrm{a}}]_n$ is shown as a function of inverse of peak sunspot number of nth solar cycle, $[S_{\mathrm{Max}}]_n$ for Group (a) I, (b) II and (c) III, where $n = 1-23$ is the solar cycle number.

a function of $[L]_n/[S_{\mathrm{Max}}]_{n+1}$ for Solar Cycles $1-22$, separately for Group I, II and III. The correlation coefficients between $[T_{\mathrm{a}}]_{n+1}$ and $[L]_n/[S_{\mathrm{Max}}]_{n+1}$ are found to be 0.92, 0.94 and 0.85 for Group I, II and III, respectively. Least square fitting of $[T_{\mathrm{a}}]_{n+1}$ with $[L]_n/[S_{\mathrm{Max}}]_{n+1}$ provides the following three equations for the Groups I – III:

$$\text{Group I:}\quad [T_{\mathrm{a}}]_{n+1} = 20.01 \times \frac{[L]_n}{[S_{\mathrm{Max}}]_{n+1}} + 2.42, \tag{11}$$

$$\text{Group II:}\quad [T_{\mathrm{a}}]_{n+1} = 21.55 \times \frac{[L]_n}{[S_{\mathrm{Max}}]_{n+1}} + 2.33, \tag{12}$$

$$\text{Group III:}\quad [T_{\mathrm{a}}]_{n+1} = 12.64 \times \frac{[L]_n}{[S_{\mathrm{Max}}]_{n+1}} + 3.29. \tag{13}$$

The above equations give the relation between the ascent time and the peak sunspot number of the same solar cycle with the parameter L of the corresponding preceding solar cycle. These equations can be used to forecast the ascent time of an upcoming solar cycle only if the prediction of the peak sunspot number of that upcoming solar cycle is known. The estimated parameter $[Q_{\mathrm{C}}]_{23}$ for Group I provides a forecast of 74($\pm$10) for the peak smoothed sunspot number for Solar Cycle 24. By utilizing this prediction of the peak sunspot number for Solar Cycle 24, *i.e.* $[S_{\mathrm{Max}}]_{24} = 74$ and the estimated quantity $[L]_{23} = 9.28$, in Equation (11), the forecast of the ascent time for Solar Cycle 24 comes out as 4.9 ± 0.4 years.

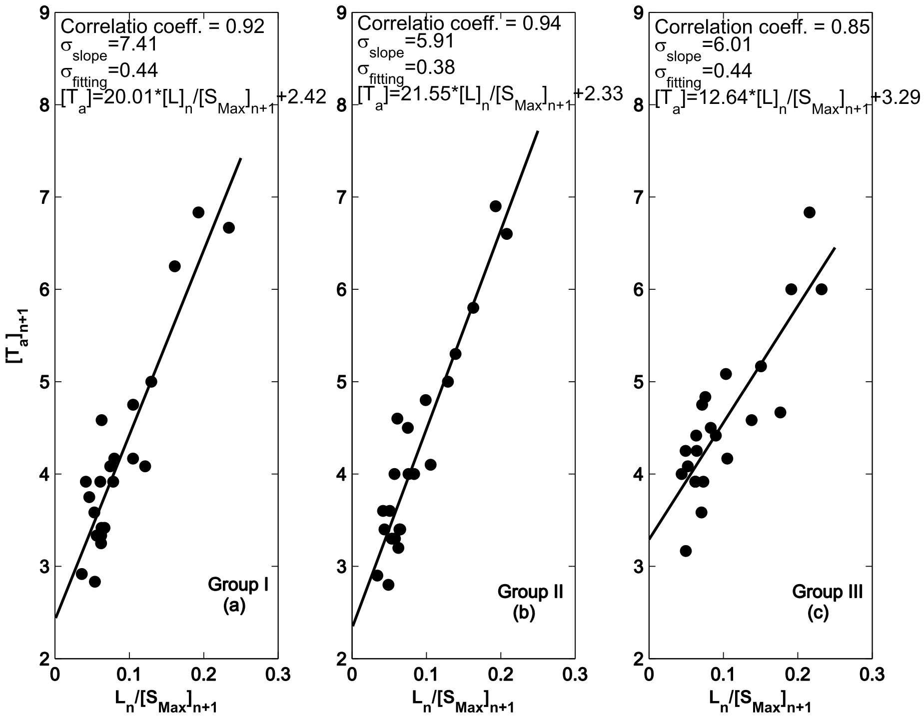

Figure 6 Ascent time for $n+1$th solar cycle, $[T_\mathrm{a}]_{n+1}$ is plotted as a function of quantity $[L]_n/[S_\mathrm{Max}]_{n+1}$ for Group (a) I, (b) II and (c) III, where $n = 1-22$ is the solar cycle number.

4. Discussion and Conclusions

In the present study, a precursor based new technique is proposed to forecast the peak sunspot number and the ascent time of the upcoming solar cycle. This method essentially utilizes the two parameters $[Q_\mathrm{C}]$ and $[L]$, which are derived numerically solely from the information of the preceding solar cycle. These parameters are computed for each nth solar cycle using "Monthly" and "Monthly smoothed" data of the international sunspot number produced by SIDC. The solar cycle characteristics defined using three different methods are compiled in Group I (Mathematical), II (Website) and III (Gaussian filter smoothing). Through a similar mathematical treatment, Q_C and L are determined for Solar Cycles 1–23, using solar cycle characteristics of Group I, II and III separately. The dependence of the peak sunspot number of the $n+1$th solar cycle on the parameter Q_C of the nth solar cycle is investigated for each of these groups. In a recent study by Kane (2008), ascent time, descent time and length of the previous solar cycle is compared with the maximum amplitude of the following solar cycle. In that study the author found that the length of the previous solar cycle and the peak of the next solar cycle are anticorrelated. Also, from past studies it is known that the peak sunspot number varies inversely with the ascent time of same solar cycle (Waldmeier, 1935). However, in a recent study it is found that the anticorrelation between peak sunspot number and ascent time reduces significantly if the "Group" sunspot number constructed by Hoyt and Schatten is used instead of the international sunspot number (Hathaway, Wilson, and Reichmann, 2002; Hoyt and Schatten, 1998). In the same context the relation between the ascent time of the $n+1$th solar cycle and the quantity $[L]_n/[S_\mathrm{Max}]_{n+1}$

Table 3 Summary of the results obtained from the present study. Correlation coefficient between *i*) peak sunspot number, $[S_{\mathrm{Max}}]_{n+1}$ and the quantity $[Q_{\mathrm{C}}]_n$, *ii*) ascent time, $[T_{\mathrm{a}}]_{n+1}$ and $[L]_n/[S_{\mathrm{Max}}]_{n+1}$ and the equations obtained from least square fitting for Groups I – III.

Relation between $[S_{\mathrm{Max}}]_{n+1}$ and $[Q_{\mathrm{C}}]_n$		
Group.	Corr. Coefficient	Equation
I	0.97	$[S_{\mathrm{Max}}]_{n+1} = 3.06 \times [Q_{\mathrm{C}}]_n + 9.06$
II	0.93	$[S_{\mathrm{Max}}]_{n+1} = 3.23 \times [Q_{\mathrm{C}}]_n + 8.89$
III	0.88	$[S_{\mathrm{Max}}]_{n+1} = 2.33 \times [Q_{\mathrm{C}}]_n + 3.26$
Relation between $[T_{\mathrm{a}}]_{n+1}$ and $[L]_n/[S_{\mathrm{Max}}]_{n+1}$		
Group.	Corr. Coefficient	Equation
I	0.92	$[T_{\mathrm{a}}]_{n+1} = 20.01 \times [L]_n/[S_{\mathrm{Max}}]_{n+1} + 2.42$
II	0.94	$[T_{\mathrm{a}}]_{n+1} = 21.55 \times [L]_n/[S_{\mathrm{Max}}]_{n+1} + 2.33$
III	0.85	$[T_{\mathrm{a}}]_{n+1} = 12.64 \times [L]_n/[S_{\mathrm{Max}}]_{n+1} + 3.29$

is also examined for Group I – III separately. The correlation coefficient between *i*) $[S_{\mathrm{Max}}]_{n+1}$ and $[Q_{\mathrm{C}}]_n$, *ii*) $[T_{\mathrm{a}}]_{n+1}$ and $[L]_n/[S_{\mathrm{Max}}]_{n+1}$ and the relations obtained by least square fitting of these parameters are provided in Table 3 for comparison.

It is noticed that $[S_{\mathrm{Max}}]_{n+1}$ is in good correlation with the parameter $[Q_{\mathrm{C}}]_n$ for Groups I – III. For all three groups, the correlation coefficient between $[S_{\mathrm{Max}}]_{n+1}$ and $[Q_{\mathrm{C}}]_n$ is found to be greater than 0.87. Nevertheless, the highest correlation coefficient of 0.97 is obtained for Group I, which is comparable to the correlation coefficient given by other precursor based prediction techniques discussed in Section 1. It is observed that the ascent time of the $n+1$th solar cycle is correlated well with the quantity $[L]_n/[S_{\mathrm{Max}}]_{n+1}$ and gives a correlation coefficient of 0.92 for Group I. It should be remarked that the correlation coefficient obtained using the proposed method is based on data of Solar Cycles 1 – 22, unlike geomagnetic precursor method which utilizes data of Solar Cycles 12 – 22. The empirical relation between $[Q_{\mathrm{C}}]_n$ and $[S_{\mathrm{Max}}]_{n+1}$ for Group I mentioned in Table 3 can be used for a prediction of the peak sunspot number of Solar Cycle 24. However, the relation between $[T_{\mathrm{a}}]_{n+1}$ and $[L]_n/[S_{\mathrm{Max}}]_{n+1}$ can be used for the prediction of the ascent time of Solar Cycle 24 only if the forecast of the peak sunspot number for Solar Cycle 24 is available. For Solar Cycle 23 a mathematical minimum of 1.7 was reached in November 2008. Hence the estimates of L and Q_{C} of Solar Cycle 23 are available only for Group I. The value computed, $[Q_{\mathrm{C}}]_{23} = 21.14$, yields a peak sunspot number of 74 for Solar Cycle 24. Similarly, the forecast for the ascent time for Solar Cycle 24 comes out as 4.9 ± 0.4 years, by considering the predicted value $[S_{\mathrm{Max}}]_{24} = 74$. So the proposed method predicts a peak of 74 ± 10 in the smoothed sunspot number close to mid October 2013.

A study using the calibrated flux-transport dynamo model forecasts a stronger peak for Solar Cycle 24 (Dikpati, de Toma, and Gilman, 2006). Another study, which is based on a method described by Feynman (1982), suggests a peak of 160 ± 25 for Solar Cycle 24 (Hathaway and Wilson, 2006). Despite the application of different methods by Hathaway *et al.* and Dikpati *et al.* in the above-mentioned works, their results suggest a stronger peak for Solar Cycle 24. There are other studies which forecast a peak sunspot number greater than 100 for Solar Cycle 24 (Bhatt, Jain, and Aggarwal, 2009; Volobuev, 2009; Hiremath, 2008; Lantos, 2006). The method used in the present study gives an estimate of 74 ± 10 for the peak of Solar Cycle 24, which contradicts those predictions. Ohl's method (1966), based on the *aa* index, indicates a peak of 70 ± 18 for Solar Cycle 24. This prediction is available at http://solarscience.msfc.nasa.gov/predict.shtml and it is close to the predictions given by the method proposed in the present work. The prediction of the peak of Solar Cycle 24 obtained using the method proposed in the present study is also in agreement with the prediction given by Javaraiah (2007), using Greenwich and SOON sunspot group data. The dynamo based model predicts that Cycle 24 will be about 35% weaker than Cycle 23 (Choudhuri, Chatterjee, and Jiang, 2007). Recently, 50 different methods for the prediction of Solar Cycle 24 have been summarized by Pesnell (2008). There are other earlier investigations using the polar field, which suggest a peak smoothed sunspot number in the range of 75 – 80 for Solar Cycle 24 (Schatten, 2005; Svalgaard, Cliver, and Kamide, 2005). However, there are some other investigations which predict a slightly higher value for Solar Cycle 24 as compared to the value predicted by the present method. A recent study by Yoshida and Yamagishi (2010) gives an estimate of 84.5 ± 23.9 for the peak sunspot number of Solar Cycle 24. Brajša *et al.* (2009) used the method of minimum and maximum and suggested a peak sunspot number in the range of 67 – 81 for Solar Cycle 24. A recent study provides two estimates; 65 ± 16 and 87 ± 13 of the peak sunspot number for Solar Cycle 24 (Aguirre, Letellier, and Maquet, 2008). The recent study by Kane (2010), using a minimum *aa* index of 8.7 centered at May 2009, predicts a peak sunspot number of 58 ± 25 for Solar Cycle 24, which is lower than the value predicted by the present method.

The relation between $[T_\mathrm{a}]_{n+1}$ and $[L]_n/[S_\mathrm{Max}]_{n+1}$ suggests that the ascent time varies inversely with the peak sunspot number of the same solar cycle. Many dynamo models for solar cycle studies have shown that the stronger cycle has a tendency to rise faster to their maximum activity (Stix, 1972; Hoyng, 1993; Ossendrijver and Hoyng, 1996; Cameron and Schüssler, 2007). However, it is observed that the ascent time T_a also depends on the parameter L of the previous solar cycle. The parameter L basically contains information about the ascending phase and the length of a solar cycle. Equation (11) can be used for the prediction of the ascent time of upcoming solar cycle if the prediction of the peak sunspot number for that upcoming solar cycle is available. Many statistically based models proposed in the past provide a reliable prediction of the peak sunspot number for the next solar cycle. As the correlation between $[Q_\mathrm{C}]_n$ and $[S_\mathrm{Max}]_{n+1}$ is found to be 0.97 with a confidence limit of more than 99%, it is suggested that the method proposed in the present study can be used for forecasting of the peak sunspot number. Similarly the correlation between $[T_\mathrm{a}]_{n+1}$ and $[L]_n/[S_\mathrm{Max}]_{n+1}$ is sufficiently high that one can use their relation for prediction of ascent time of the $n+1$th solar cycle, provided that the prediction of the peak sunspot number for the $n+1$th solar cycle is known. The disadvantage of this method is that the quantities Q_C and L cannot be computed for a previous solar cycle unless the mathematical minimum is reached in the monthly smoothed sunspot number.

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