

# Are the equatorial electrojet and the Sq coupled systems? Transfer entropy approach

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## Abstract

Whether equatorial electrojet (EEJ) and solar quiet (Sq) are independent systems or not is a long standing question. Techniques such as correlation analysis, interpretation of the westward currents observed between EEJ and Sq focus, along with the simulation studies have been used to address this question, hitherto. In this article, we revisit this problem using a method based on transfer entropy that examines the relationship between day-to-day variability in EEJ and Sq during low solar activity period (year 2007–08). Magnetic field variations in the horizontal component from the geomagnetic observatory, Tirunelveli (TIR) from the Indian region are used as a proxy for EEJ currents. To represent variations of Sq current system, two stations outside the EEJ belt, Nagpur (NGP) and Jaipur (JAI) are analyzed. Our analyses clearly demonstrate that significant information is exchanged between EEJ and Sq variations, and hence they are in a cross-talk with each other, indicating EEJ and Sq are coupled systems. Variations of time scales less than 2 h appear at the equatorial station before Sq stations. Similar analyses carried out for the African sector also validate the above results.

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## 1. Introduction

The electric currents flowing in the dayside E-region are essentially produced by the ionospheric dynamo due to movement of charged particles (through their collisions with neutral wind) across the Earth's magnetic field. The normal solar quiet day (Sq) variation of geomagnetic field observed globally during sunlit hours is associated with two equivalent current loops centered around the focal points at  $\sim \pm 30^\circ$  latitudes, located in each daytime hemisphere (Chapman and Bartels, 1940). In the northern hemisphere, the current vortex is counter clockwise and it is clockwise in the southern hemisphere. Near the dip equator very large eastward currents flow in a narrow strip, are

called Equatorial electrojet (EEJ) (Chapman, 1951). EEJ index that represents the magnitude of the electrojet currents is computed by taking difference in the horizontal component of the magnetic field between two stations: one near dip equator and other just outside the equatorial belt, in order to eliminate the contribution of Sq from the equatorial station. Several ground based as well as satellite based studies are carried out to understand the characteristics of Sq and EEJ current systems (Cain and Sweeney, 1973; Onwumechili, 1992; Jadhav et al., 2002a,b; Gurubaran, 2002; Lühr et al., 2004; Rastogi et al., 2004; England et al., 2006; Manoj et al., 2006; Yamazaki et al., 2010; Rabiou et al., 2013; Hamid et al., 2014, and many more). Various general circulation models such as NCAR's Thermosphere-Ionosphere General Circulation Model (TIE-GCM) (Richmond et al., 1992), Coupled Thermosphere-Ionosphere Model (CTIM) (Fuller-Rowell et al., 1996), Coupled Thermosphere-Ionosphere-Plasmasphere

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Model (CTIP) (Millward et al., 1996), Kyushu-GCM (Yoshikawa and Miyahara, 2003) and Global Ionosphere Thermosphere Model (GITM) (Vichare et al., 2012) reproduce the ionospheric dynamo including the EEJ successfully.

An important question that is debated for long is whether EEJ is a part of the global Sq current system showing an additional enhancement near the equator due to the spatial geometry of the geomagnetic field lines near the dip equator (Baker and Martyn, 1953), or these two are totally different current systems without any connection between them. Many attempts have been made to address this question for several decades (Onwumechili, 1997, and references therein).

For a divergence-less current system, current flows in a closed circuit. In case EEJ and Sq are two unconnected current systems then they are expected to complete their circuit in an independent manner. Sq current system consists of two complete current loops, but there is no such clear evidence of closed currents related to the EEJ using raw ground magnetic field data. However, all low earth orbiting (LEO) satellite missions have clearly shown the existence of westward currents at the flanks of the EEJ (Cain and Sweeney, 1973; Jadhav et al., 2002a; Lühr et al., 2004). It is possible that eastward EEJ current can complete its path in a horizontal plane with westward return currents flowing at some latitudes between the dip equator and Sq foci (Onwumechili, 1992). However on ground, the superimposed eastward Sq currents can mask these westward currents and hence difficult to detect the return currents of EEJ in the ground magnetic records. Removal of background Sq can elicit the actual latitudinal profile of EEJ. In a comprehensive review on return currents of EEJ, Onwumechili (1992) employed model (empirical and theoretical models) and observations from various platforms (such as ground, satellite, and rocket) to document presence of return current at the edges of EEJ in both hemispheres. He concluded that return currents are permanently connected to the EEJ and are complete in nature, suggestive of a balance between the westward return and eastward EEJ currents. This supports the idea of existence of separate EEJ current system. However, Stening (1995) reexamined the above proposition and concluded that EEJ is an integral part of Sq, with superposed current systems driven by other tidal modes. He argued that the eastward and westward currents are driven by different sources and hence are not related to each other. Various models have shown that the vertical wind shears associated with tidal winds and gravity waves can generate westward currents (particularly beyond 3° geomagnetic latitudes) with large height and latitudinal gradients, altering the intensity and latitudinal structure of the EEJ (Richmond, 1973; Reddy and Devasia, 1981; Stening, 1995). Therefore, based on the interpretation of these westward currents, there exist two schools of thought: one group of researchers consider the westward currents as return currents of EEJ, while others who do not subscribe to the above view ascribe their

origin to local winds, thus suggesting EEJ and Sq to constitute a single current system.

Besides these interpretations, there exists other studies which employed correlation techniques to examine relationship between these two current systems. In case EEJ is a part of the global Sq current system, the day-to-day variations in EEJ would correlate well with those outside the EEJ belt. Forbush and Casaverdet (1961) however found that the daily variations in the EEJ region were independent of the Sq variations outside EEJ belt. Several other researchers also reported no correlation between EEJ and Sq (e.g. Rangarajan and Murty, 1984). Recently, Manoj et al. (2006) compared EEJ estimates obtained from CHAMP satellite with the ground observations outside the EEJ belt and concluded that variations in EEJ and Sq are unrelated. While studying the relation between day-to-day variability of EEJ and neutral wind using TIME-GCM simulations, Yamazaki et al. (2014) found that the dominant mechanism for the day-to-day variability can be different for EEJ and Sq. They observed that the correlation between the daily range of Sq at the magnetic equator and other latitudes separated by 10° drops significantly. On the other hand, Yamazaki et al. (2010) found that Sq and EEJ correlate well on a long-term basis, reacting in the same manner to temporal changes of solar ionization and heating of the ionosphere. Kane (1971) also reported a good correlation between the two, when corrected for Disturbance storm time index. The influence of common factors may result in a good correlation without any bearing on the actual dependency. Also, poor correlation can result due to superposed current systems driven by different tidal modes (Stening, 1995). Therefore, to understand the dynamic nonlinear interactions between these two current systems, correlation analysis may not be adequate. Recently Hamid et al. (2014) demonstrated that the conclusions regarding the relationships between EEJ and Sq, based on the correlations vary significantly with the field representing EEJ i.e. EEJ index (two-station difference) or H-variations at the equatorial station.

State of the art numerical general circulation ionospheric models compute electric fields and currents self-consistently. These models facilitate to study the variability in the ionospheric currents flowing at equatorial and Sq locations due to the forcing from below, and can help to understand the connection between EEJ and Sq. Attempts have been made to investigate the role of various tidal modes of upward propagating migrating, non-migrating tides, gravity and planetary waves in the ionospheric electrodynamics (Richmond et al., 1976; Du and Stening, 1999; Pogoreltsev et al., 2007; Kawano-Sasaki and Miyahara, 2008; Fang et al., 2008; Liu et al., 2010). All these works demonstrate that atmospheric waves drive EEJ and Sq currents and their variability. Further, Maus et al. (2007) inverted CHAMP satellite observations of EEJ profiles using the solutions of the governing electrodynamic equations to obtain the estimates of the eastward electric fields. Also Doumbia et al. (2007) examined

whether the magnetic variations obtained by TIE-GCM simulations can reproduce the major features of the equatorial electrojet (EEJ) as observed on the ground as well as onboard low-altitude orbiting satellites. Thus, attempts have been made in linking theory to the vast body of observations.

Methodologies based on information transfer have been effectively applied in diverse fields including neuroscience, climate change, magnetospheric dynamics, solar physics (Vicente et al., 2011; Das Sharma et al., 2012; De Michelis et al., 2011; Johnson and Wing, 2014; Kakad et al., 2015) besides many other research areas to establish the inter-connectivity and recognise prominent drivers in coupled systems. In this article, we revisit the problem of coupling between Sq and EEJ current systems by applying information theory-based stochastic methods that revolve around the concept of entropy or ensemble information content (Shannon, 1948). The magnetic field variations in the horizontal component recorded at an equatorial observatory contain major contribution from the EEJ currents while those recorded at low latitude stations located outside the EEJ belt can be considered as a proxy to Sq variations. As a first step, entropy based quantitative estimates of similarity such as mutual information (MI) which are non-metric, and metric measures like normalized information distance (NID) are employed to map the informatic structure of the current systems. This is followed by estimation of transfer entropy (TE) between the two proxy variables that represent EEJ and Sq currents to measure the flow of information from EEJ to Sq and Sq to EEJ systems. The primary advantage of transfer entropy is that it disregards the effects of common drivers occurring simultaneously while mapping the transfer of information between any two time series representative of the phenomena or physical processes. Thus, use of this method in our study ignores the influence of common forcing factors such as F10.7, Solar UV flux, solar flare, ring current or magnetospheric disturbances etc, which affect both EEJ and Sq in similar fashion simultaneously. Finally, utilizing data from the Indian and African sectors we demonstrate that EEJ and Sq are indeed coupled systems.

## 2. Data used and processing

In the present work, we use geomagnetic field variations of one minute sampling interval recorded at three Indian magnetic observatories during low solar activity period in 2007–08. The geographic and geomagnetic coordinates of

these observatories are shown in Table 1. The magnetic observatory Tirunelveli (TIR), is located near the dip equator and hence the recorded magnetic field variations at this location contain major contribution due to the overhead equatorial current system and therefore represents EEJ current. The observatories at Nagpur (NGP) and Jaipur (JAI) are situated at low latitude and lie outside the EEJ belt. Hence data from these can be considered as a proxy to Sq variations. Note that these two Sq stations generally lie between the dip equator and Sq focus, experiencing predominantly the eastward current flow of the Sq current loop in the northern hemisphere. All the three selected stations fall in a narrow belt of longitude minimizing any local time differences. The data gaps are interpolated using cubic Hermite polynomial for uniformity of the time series. To eliminate the edge effects, we have eliminated 60 data points from both ends of the interpolated time series. Fig. 1 shows the original time series recorded at all three Indian stations during 2007–08. Note that we have deliberately added vertical offsets in the time series shown in Figs. 1–3 to avoid the overlaps.

The information theory-based stochastic methods should be applied to the randomly varying time series. Therefore, it is necessary to transform the non-stationary time series to stationary or stochastic sequence. As a contrast to non-stationary series, stationary series shows constant mean and variance over time. The removal of deterministic trends transforms the time series into stationary one (Wei, 1994; Carbone et al., 2004; Bossomaier et al., 2013). Therefore, this can be achieved by removing the moving average from the original series, which acts like a high pass filter (Carbone et al., 2004). The moving average of time series,  $\bar{x}_n(t)$  is estimated with the help of following formula:

$$\bar{x}_n(t) = \frac{1}{n} \sum_{k=0}^n x(t-k) \quad (6)$$

where  $n$  is the size of the moving window.

In order to avoid the under and over smoothing of the data, it is required to examine the stationary time series carefully (Das Sharma et al., 2012). The stationarity or the randomness of the series can be monitored through the Gaussian type probability distribution. Therefore we use Gaussian distribution as a selection criterion for the optimum window size of moving average. Further, to find the probability distribution, a histogram technique is used. Choosing an appropriate bin size is one of the important things in histogram analysis. If one uses a large bin size

Table 1  
Magnetic observatories from Indian sector.

| Magnetic Observatory | Code | Geographic    |                | Geomagnetic   |                |
|----------------------|------|---------------|----------------|---------------|----------------|
|                      |      | Latitude (°N) | Longitude (°E) | Latitude (°N) | Longitude (°E) |
| Tirunelveli          | TIR  | 8.70          | 77.80          | −0.17         | 150.40         |
| Nagpur               | NGP  | 21.15         | 79.08          | 12.33         | 152.71         |
| Jaipur               | JAI  | 26.92         | 75.80          | 18.35         | 150.16         |

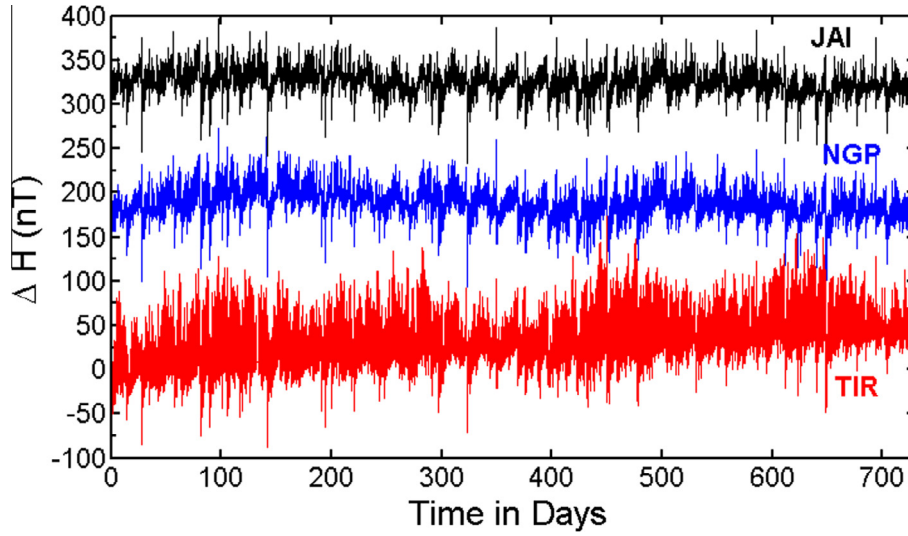


Fig. 1. Magnetic field variations in H component at three stations during two years (2007–08). Vertical offsets are added to NGP and JPR for better presentation of the data.

the histogram becomes too smooth and some information can be lost. While use of a smaller bin width results in a histogram that is rough with many empty bins and might not reveal the true nature of the underlying probability distribution. An optimum bin width is obtained applying the Scott's law,  $\omega = 3.49\sigma N^{1/3}$  (Scott, 1979) where  $\omega$  is the optimum bin width,  $N$  is number of data points and  $\sigma$  is standard deviation of the series. We estimated the optimum bin width and corresponding probability distributions for each series

In Fig. 2, we have demonstrated the selection of a moving window size using typical values  $n = 3, 5, 7$  and  $13$ . Note that Fig. 2 displays the magnetic field variations (original and running means) during 20 days for the better visibility, but the histograms of probability distribution are for entire two years period. The distributions using  $n = 3$  and  $5$  result in good Gaussian type distribution and hence confirms the stochastic nature of the filtered series. However, the moving average with  $n = 3$  mimics the original time series almost entirely and hence under-smoothes the time series. The distributions using  $n = 7$  and  $13$  are non-Gaussian, particularly for TIR station and hence are not suitable for the present study. Thus, moving window size,  $n = 5$ , is found optimal to transform the recorded time series into a stationary sequence.

Fig. 3 shows the stationary time series obtained after subtracting the moving average of window size 5 and are suitable for further statistical analysis.

We estimate the MI and time-delayed information flow between EEJ and Sq stations to understand the connection between the two. To evaluate the statistical significance of the obtained results, significance level of the MI and TE were estimated following surrogate data test. To investigate the dependence of TE, if any, on different temporal resolutions we also estimated the entropy values at different time resolutions, starting from 30 min to 120 min.

### 3. Estimation of mutual information and transfer entropy

Information theory is based on the concept of Entropy, which is a measure of the uncertainty associated with a random variable. Recognizing that the concepts of uncertainty and information are related, Shannon (1948) realized that the occurrence of an  $i$ th event having lower probability ( $P_i$ ) implies more information. Based on this understanding, a probabilistic interpretation of entropy (degree of randomness) called Information Entropy or Shannon entropy,  $H$  is defined for a random variable  $x$  as:

$$H_x = - \sum_{i=1}^N P_i(x) \log_2 \left( \frac{1}{P_i(x)} \right) \quad (1)$$

Note that,  $H_x$  is a function of the probability distribution of random variable  $x$ .

Mutual information (MI) is a measure of the amount of information that one random variable say,  $X$  contains about another random variable,  $Y$ . Therefore, MI between two random distributions  $x$  and  $y$  quantifies the shared information between them and it is defined as:

$$MI_{xy} = H_x + H_y - H_{x,y} \quad (2)$$

where  $H_{x,y}$  is the joint entropy, which denotes the total information content of the two variables  $x, y$ .

In terms of probabilities, MI can be written as

$$MI_{xy} = - \sum_{i,j=1}^N P_{ij}(x,y) \log_2 \left( \frac{P_{ij}(x,y)}{P_i(x) P_j(y)} \right) \quad (3)$$

where  $P_{ij}(x,y)$  is the joint probability of  $x$  and  $y$ , while  $P_i(x)$  and  $P_j(y)$  are probabilities of observing  $x$  and  $y$  independently. By using chain rule, the joint probability of the occurrence of  $x$  and  $y$  could be expanded as

$$P(x,y) = p(y)p(x|y) \quad (4)$$

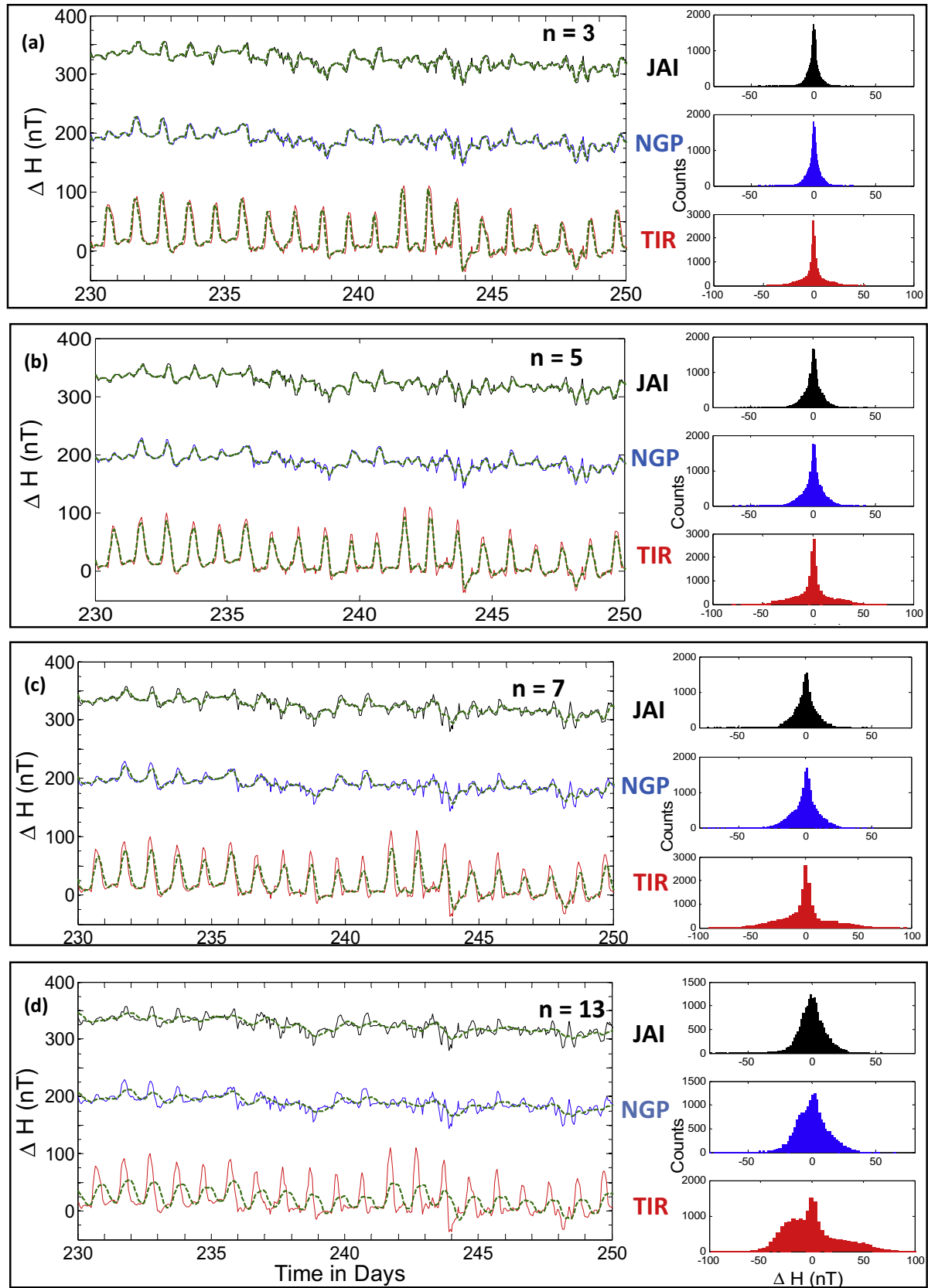


Fig. 2. For a moving window size of (a)  $n = 3$ , (b)  $n = 5$ , (c)  $n = 7$ , (d)  $n = 13$ . Left panel: original (solid) and smoothed (dashed)  $H$  variations; Right panel: histograms.

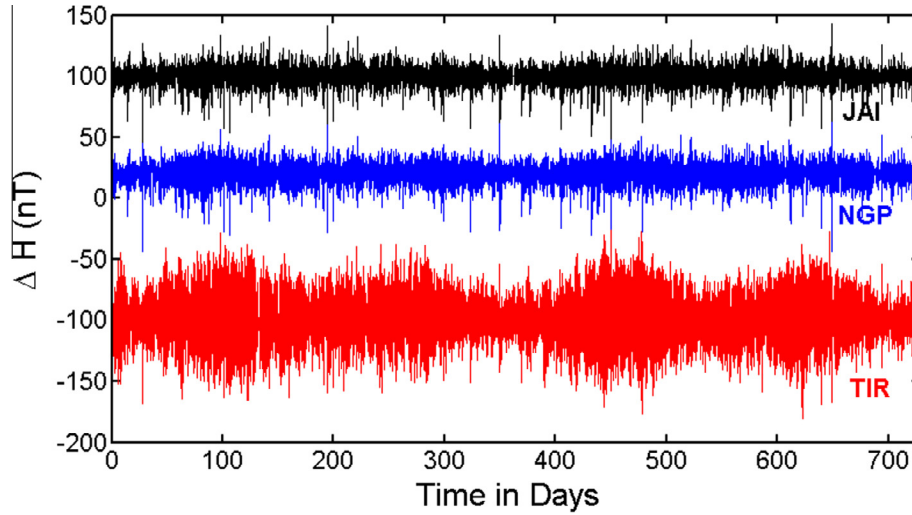


Fig. 3. Stationary time series at three stations.

where  $P(x|y)$  is conditional probability which is the probability of the occurrence of event  $x$ , when  $y$  occurs. For three variables joint probability is

$$P(x, y, z) = P(x|y, z) P(y, z) = P(x|y, z) P(y|z)P(z)$$

Note that MI is the reduction in the uncertainty of  $x$  due to the knowledge of  $y$ . A variant of MI that takes into account a time delay  $\tau$  between the two signals, named as delayed mutual information  $MI_{xy}(\tau)$  is analogous to the cross-correlation function but with nonlinearity in correlation being incorporated into this measure. The time delayed MI can be estimated using:

$$MI_{xy}(\tau) = \sum_{i,j=1}^N P_{ij}(x(t), y(t+\tau)) \log_2 \left( \frac{P_{ij}(x(t), y(t+\tau))}{P_i(x(t)) P_j(y(t+\tau))} \right) \quad (5)$$

where  $P(y(t+\tau), x(t))$  is the joint probability of  $y(t+\tau)$  and  $x(t)$ .

However, it is pertinent to note that while MI is symmetric ( $MI_{xy} = MI_{yx}$ ) and therefore lacks directionality, the delayed MI cannot distinguish between actually exchanged information and shared information due to common input or history. As both these measures are non-normalized and non-metric, it is not possible to compare the similarities in the data cluster (refer supplementary material of [Das Sharma et al., 2012](#)). Therefore, we define ‘Normalized Information Distance’ (NID), which uses the maximum of the two entropies ( $\max\{H(x), H(y)\}$ ) as a normalization factor and is given as:

$$NID(x, y) = 1 - \frac{MI_{xy}}{\max\{H_x, H_y\}} \quad (6)$$

Yet, the above mutual information based measures do not contain directional information between any two variables ( $x, y$ ). An alternative theoretic information measure known as transfer entropy (TE) introduced by [Schreiber \(2000\)](#)

overcomes the cited difficulties and explicitly quantifies actual information exchanged along with the directional flow of information between any two variables ([Schreiber, 2000](#); [De Michelis et al., 2011](#)). The TE from a process represented by variable  $x$  to another process  $y$  after a time lag  $\tau$  is the quantity of information that the state of  $y$  has at a time  $(t + \tau)$  based exclusively on the state of  $x$  at time  $t$ . Therefore, transfer entropy between two random variables or processes  $x$  and  $y$  is mathematically represented as:

$$TE_{x \rightarrow y}(\tau) = \sum P(y(t+\tau), y(t), x(t)) \log_2 \left( \frac{P(y(t+\tau), y(t), x(t)) * P(y(t))}{P(x(t), y(t)) * P(y(t+\tau), y(t))} \right) \quad (7)$$

where  $P(y(t+\tau), y(t), x(t))$  is the joint probability of  $y(t+\tau)$ ,  $y(t)$  and  $x(t)$ .

Similarly, one can compute the transfer entropy from  $y$  to  $x$ ,  $TE_{y \rightarrow x}(\tau)$  with time delay  $\tau$ . Thus, transfer entropy gives a measure of exchanged information between two time series, representing two physical systems. Note that with the exchange of variables  $x$  and  $y$ , the equation results in different values implying asymmetry in the information transfer between the two variables. TE exhibits following properties: (a) measure of information transferred (exchanged) rather than information shared, (b) asymmetric measure of information flow, (c) enables quantifying information flow separately in both directions, (d) incorporates the directionality of net information flow and (e) is a model independent measure of causality ([Das Sharma et al., 2012](#)). Note that asymmetry of transfer entropy under exchange of  $x$  and  $y$  gives rise to a net flow of information between the two representative time series. If  $TE_{x \rightarrow y}(\tau) > TE_{y \rightarrow x}(\tau)$  then one can infer that physical process represented by time series  $x$  influences  $y$ . According to causality principle, there exist finite time lag between source and response.

#### 4. Results

We have computed the mutual information and the time delayed variant between TIR and each Sq station. In Fig. 4 time delayed mutual information (TDMI) between EEJ and Sq stations is shown. The left hand side plots show the TDMI between time delayed equatorial time series TIR ( $t + \tau$ ) and Sq stations JAI( $t$ ) and NGP( $t$ ). While, right side plots show TDMI between time delayed series of Sq stations (JAI( $t + \tau$ ) and NGP( $t + \tau$ )) and TIR( $t$ ). The values for all above combinations are estimated using time series of hourly sampling interval. Applying surrogate data test, we have estimated the 5% significance level threshold following Theiler et al. (1992), and is shown by dashed line in each plot. The plots show that the mutual information is clearly significant for the considered range of time delay up to 10 h. The MI shows a monotonically decreasing trend with time delay, with maxima at zero time suggesting that maximum information is shared between Sq and EEJ stations almost instantaneously, which could be due to the effect of common factors, without any influence of history on the recorded time series at hourly time-scales. A small secondary peak can be noticed at time

delays of  $\sim 4$ –8 h. Note that when second time series is shifted by 4–8 h, then the MI values resulting from the overlapping of morning sector of the first time series and afternoon sector of the second series can give augmented values, as these two sectors are anti-correlated due to nearly symmetric profiles of the diurnal variation of the Sq and EEJ fields about local noon. Since MI gives the absolute values of the non-linear correlations, this anti-correlation can result in the enhanced MI values.

In order to compare the information shared between TIR-NGP and TIR-JAI pairs, a normalized metric becomes desirable and can be obtained through NID values using Eq. (6). It is clear from this equation that smaller values of NID indicate higher shared information and vice versa. The obtained NID values are 0.7729 ( $\pm 0.0025$ ), and 0.8375 ( $\pm 0.0022$ ) for TIR-NGP and TIR-JAI pair respectively. The corresponding errors (mentioned in brackets) are estimated by applying a bootstrap analysis for 1000 realizations. This suggests that the information common between TIR-NGP is slightly higher than that between TIR-JAI pair. This could be due to closer location of NGP to TIR as compared to that of JAI.

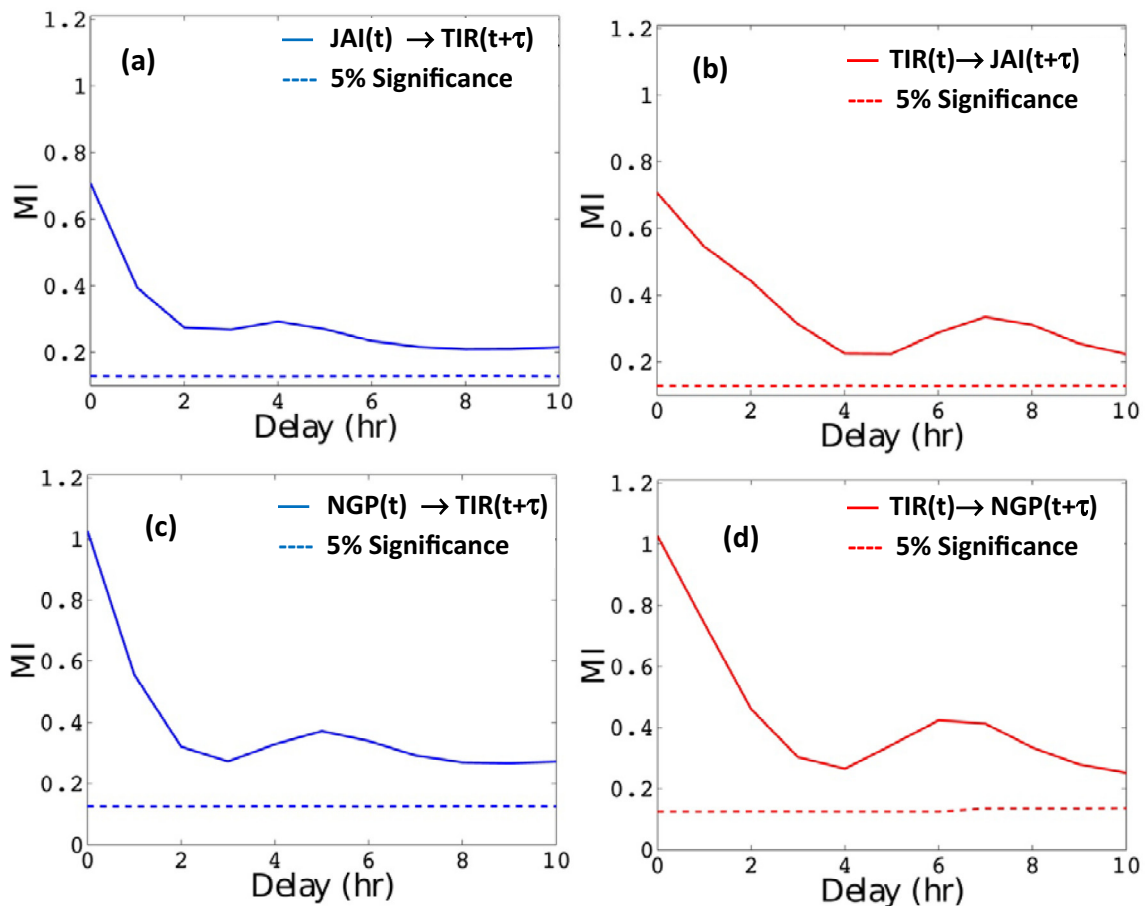


Fig. 4. Time-delayed mutual information. Left hand side top panel shows MI between time delayed series of EEJ, TIR( $t + \tau$ ) and Sq, JAI( $t$ ). Right side displays MI between JAI( $t + \tau$ ) & TIR( $t$ ). Bottom panels are same as top but for NGP station. Dashed line represents 5% significance level obtained using surrogate data that is used to generate 100 sets of uncorrelated random phase time series.

Fig. 5 shows the transfer entropy plots between EEJ and Sq locations. Left hand side panel shows the entropy transferred from Sq stations (NGP, JAI) to EEJ station TIR, while right panel shows that from EEJ location to Sq stations. The null hypothesis set up for our study is the causal relationship between the time series generated at EEJ and Sq stations. To evaluate this null hypothesis, a confidence level of 95% is set. Therefore, significance levels at 5% are computed using surrogate data test from 100 randomized datasets, and are shown by dashed curves in Fig. 5.

The directional transfer entropy ( $TE_{x \rightarrow y}(\tau)$ ) for varying time delays up to 10 h is displayed in Fig. 5. The entropy transferred from JAI  $\rightarrow$  TIR and TIR  $\rightarrow$  JAI shown in Fig. 5(a and b) is significant for time delays less than 6–8 h. The flow of information is more from TIR  $\rightarrow$  JAI than that from JAI  $\rightarrow$  TIR. The entropy transfer from TIR (EEJ station) to the other Sq station (NGP) is also above 5% significance level (Fig. 5d), but the flow from NGP to TIR is less significant (Fig. 5c).

Entropy exchange between EEJ (TIR) and Sq stations (NGP, JAI) is compared in Fig. 6. These plots demonstrate that almost throughout the entire range of time delay, the transfer entropy from EEJ to Sq stations is higher than that from Sq to EEJ station. This suggests that there is a net flow of information from EEJ to Sq stations, in the Indian sector. To gain further insights into the transfer of entropy between the EEJ and Sq stations, utilizing 5% significance

levels, a quantity known as the significant transfer entropy ( $\Delta TE_{sig}$ ) is computed (De Michelis et al., 2011).

Fig. 7 shows significant transfer entropy from the equatorial to Sq stations (red curve) and vice versa (dashed blue curve). The significant transfer entropy is defined as the difference between the transfer entropy and the 5% null hypothesis level (i.e. difference between solid and dashed curves in Fig. 5). It may be noted that positive values indicate that the exchange of information is above the significance level. The transfer entropy from TIR  $\rightarrow$  JAI is indeed significant up to time delay of  $\sim 9$  h, while that from JAI  $\rightarrow$  TIR is up to  $\sim 6$  h. For the TIR-NGP pair, though the transfer is significant, it is restricted to relatively shorter time delays of  $\sim 4$  h for TIR  $\rightarrow$  NGP and  $\sim 2$  h for NGP  $\rightarrow$  TIR. It can also be observed from Fig. 7 that in general,  $\Delta TE_{sig}$  is larger from equatorial to Sq stations. The values of  $\Delta TE_{sig}$  for TIR  $\rightarrow$  JAI are almost 3 times higher than that from TIR  $\rightarrow$  NGP. This may suggest that compared to NGP, the variations at JAI have stronger influence of those at TIR.

The peak in the significant transfer entropy from equatorial to both the Sq stations take place at a time delay of 2 h. The time delay of 2 h from equatorial to Sq station is in agreement with the maxima observed in the directional entropy transfer depicted in Fig. 5. It should be noted that the peaks in TDMI values were observed at zero time lag (Fig. 4), which is not the same as the time delay for maxima

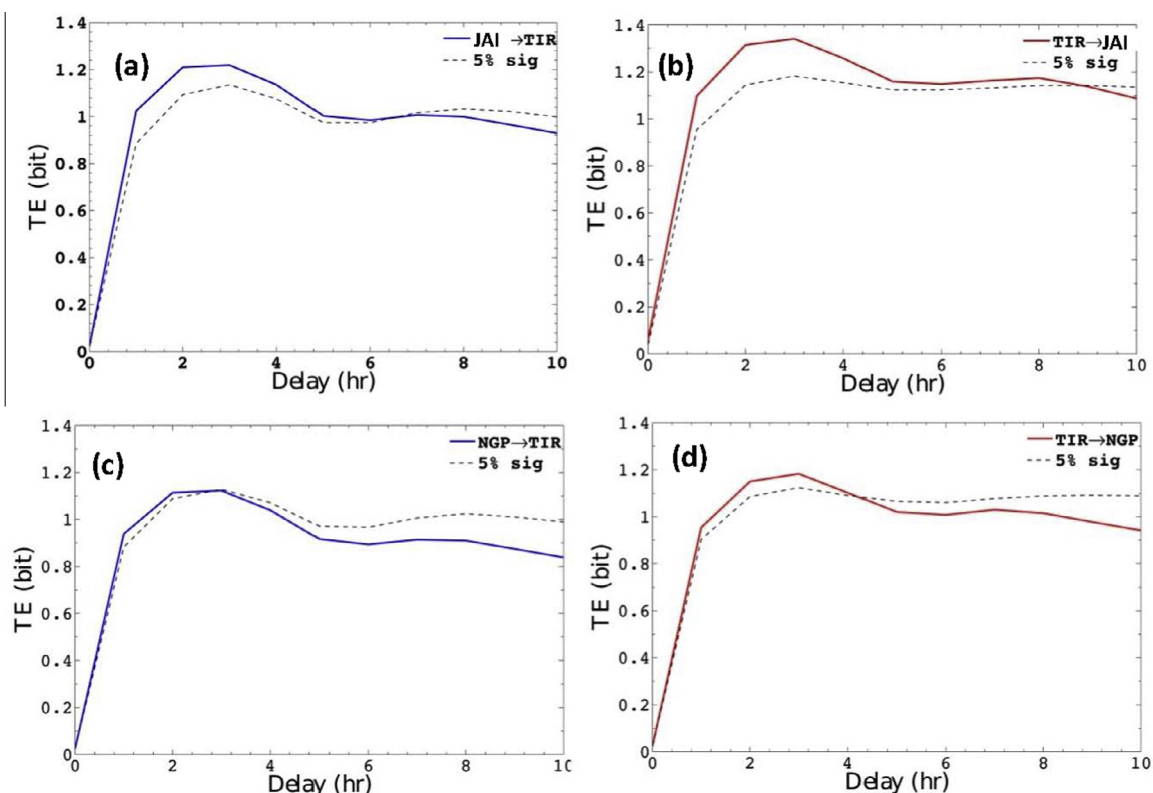


Fig. 5. The directional transfer entropy, (a) and (c) from Sq station to EEJ station (left side plots) and (b) and (d) from EEJ to Sq station (right side plots). The dashed curves are the 5% significance level, constructed from 100 surrogate datasets.

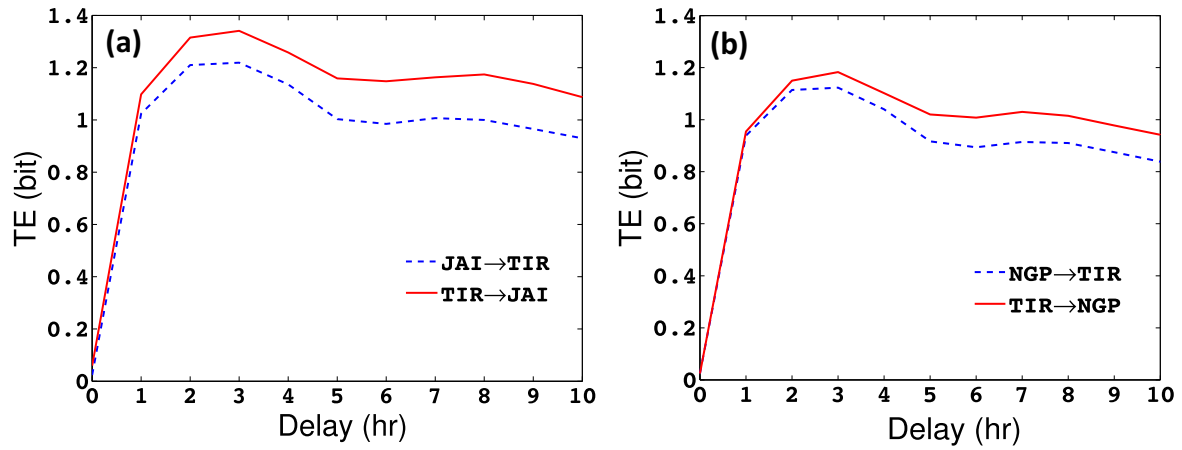


Fig. 6. Comparison of TE from EEJ to Sq station (solid red color) and Sq to EEJ station (dashed blue color). (a) JAI, (b) NGP as Sq station. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

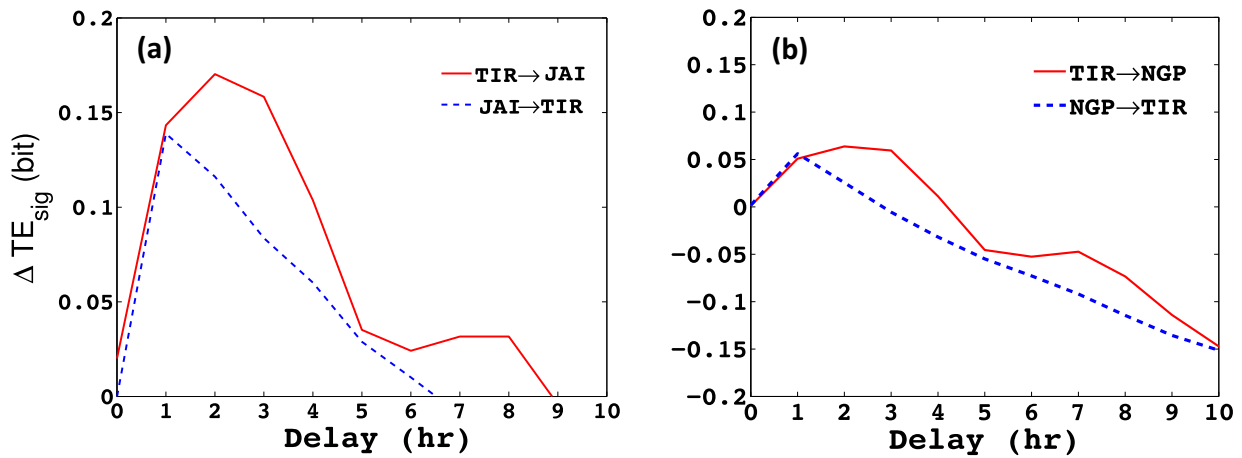


Fig. 7. Same as Fig. 6, but parameter plotted is significant transfer entropy  $\Delta TE_{sig}$ .

observed in the information flow. This has important implication that though there is a good amount of information common between equatorial and Sq stations at zero time lag, the peak in the flow of information takes place with a time lag of 2 h.

The total significant transfer entropy,  $I_{sig}^{tot}$  is computed using,  $I_{sig}^{tot} = \int_0^T \Delta TE_{sig}(t) dt$ , where  $[0, T]$  is the time interval in which  $\Delta TE_{sig} \geq 0$  (Table 2). It is found that the total significant transfer entropy for EEJ → Sq stations is 2–3 times higher compared to that from Sq to EEJ location.

The results depicted in Figs. 4–7 are for the time series with sampling interval of 1 h. To further validate our results, we also carried out a similar exercise using time series generated at varied time resolutions ranging from 30 min to 2 h, with a step of 10 min. This exercise would enable us to evaluate changes in the relationship, if any, at different time resolutions. The corresponding maxima of directional entropy transfer determined for different time resolutions are depicted in Fig. 8. The peak transfer entropy values shown for all the curves are above 5% significance level. A general decreasing trend with increasing

sampling interval is evident from this figure, which is due to averaging effect. A couple of consistent features can be noticed in Fig. 8. Firstly, the values of transfer entropy between TIR and JAI are consistently higher than the TIR–NGP pair. The other important observation is that for any Sq station, the entropy transfer from the equatorial to Sq location is always higher than in the reverse direction. These observations are consistent with the results presented earlier in this study. This confirms there is a net flow of information from the equatorial to Sq stations at time scales ranging from 30–120 min. It is also found that the transfer entropy is not significant for the data of sampling interval  $> 3$  h.

Table 2  
Total significant transfer entropy in Indian region.

| EEJ → Sq  | $I_{sig}^{tot}$ (bit × min) | Sq → EEJ  | $I_{sig}^{tot}$ (bit × min) |
|-----------|-----------------------------|-----------|-----------------------------|
| TIR → NGP | 10.824                      | NGP → TIR | 4.188                       |
| TIR → JAI | 41.55                       | JAI → TIR | 21.804                      |

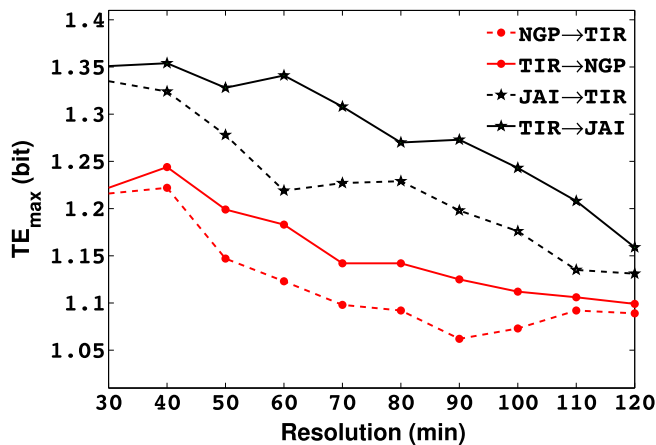


Fig. 8. Maximum value of TE plotted against the varying sampling interval of the time series.

To further vindicate our findings and conclusions, similar analysis is carried out using African zone geomagnetic data with 1 h sampling interval during the same period (2007–08). The coordinates of the three stations from Africa are presented in Table 3. Observatory Bangui (BNG) is considered to represent the EEJ (though it is slightly away by  $4^\circ$  from the geomagnetic equator, it is presumed to be in the equatorial region); and Tsumeb (TSU) and Hermanus (HER) are used as a proxy of Sq system. Note that HER lies at the poleward side of the Sq focus, and experiences westward current component of the Southern Sq loop. The geomagnetic latitude of TSU is almost similar to that of JAI, but is in the southern hemisphere. The total significant transfer entropy for these stations listed in Table 4 once again confirms that there is a significant net flow of information from EEJ  $\rightarrow$  Sq stations. However, it is noticed that the total significant transfer entropy values from EEJ  $\rightarrow$  Sq for both Sq stations are almost the same, unlike in the Indian sector.

## 5. Discussion and summary

In this paper, we adopt an innovative approach than hitherto in use to study the coupling between EEJ and Sq current systems, by finding the entropy transfer between the two which enables quantify the cross-talk, if any, between these two systems.

The geomagnetic field variations recorded at equatorial station, TIR is used as a proxy for EEJ current system, and two stations (NGP and JAI) are used to represent Sq

Table 4

Total significant transfer entropy in African region.

| EEJ $\rightarrow$ Sq  | $I_{sig}^{tot}$ (bit $\times$ min) | Sq $\rightarrow$ EEJ  | $I_{sig}^{tot}$ (bit $\times$ min) |
|-----------------------|------------------------------------|-----------------------|------------------------------------|
| BNG $\rightarrow$ TSU | 57.102                             | TSU $\rightarrow$ BNG | 37.266                             |
| BNG $\rightarrow$ HER | 57.654                             | HER $\rightarrow$ BNG | 43.908                             |

currents. The magnetic field variations at the equatorial station also contain the contribution due to Sq, which is generally removed by taking the difference between EEJ and off-equatorial station to represent EEJ. Therefore, we have re-examined the results obtained here with the EEJ represented by the above mentioned difference method. Interestingly, it is found that the results remain the same. The directional transfer entropy (which is a measure of information transferred from one station to another) reveals that the larger amount of entropy flows from EEJ  $\rightarrow$  Sq than from Sq  $\rightarrow$  EEJ station. This suggests that there is a net flow of information from EEJ to Sq station.

According to the principle of causality, the effect of the source takes finite time to reach the target. Therefore, any variations observed simultaneously at two stations are ignored. Our results show that the flow of information from EEJ  $\rightarrow$  Sq is maximum at a time delay of 2 h, indicating that the variations at EEJ station are observed at Sq station after  $\sim 2$  h. The longitudes of all the stations lie in a very narrow band, and hence possibility of the role of the local time meridian is ruled out. Due to the causality principle used in the entropy analysis, this observed time delay can imply that the variations at EEJ are causing variations in Sq. These indications do not seem to be consistent with the long accepted basic model of Sq dynamo by Baker and Martyn (1953) that describes EEJ as an enhancement of Sq near the dip equator. But our results are validated with the time series of different sampling intervals and also for the African region. We also examined the results by replacing the H variations at TIR with the time series obtained after subtracting the Sq variations from TIR (i.e. actual EEJ at TIR). All these exercises indicate the same results that there is a significant net flow of information between EEJ  $\rightarrow$  Sq and the variations appear at the equatorial station before Sq station. Hence our results are robust enough.

Also, the time delay of 2 h is observed consistently even if the data of different sampling intervals between 30 and 120 minutes are considered. At the same time, the transfer entropy is found to be non-significant for the data of time scales  $>3$  h. Note that the sampling interval of the time

Table 3  
Magnetic Observatories from African sector.

| Magnetic Observatory | Code | Geographic             |                         | Geomagnetic            |                         |
|----------------------|------|------------------------|-------------------------|------------------------|-------------------------|
|                      |      | Latitude ( $^\circ$ N) | Longitude ( $^\circ$ E) | Latitude ( $^\circ$ N) | Longitude ( $^\circ$ E) |
| Bangui               | BNG  | 4.33                   | 18.57                   | 4.1                    | 91.4                    |
| Tsumeb               | TSU  | −19.2                  | 17.58                   | −18.87                 | 85.7                    |
| Hermanus             | HER  | −34.4                  | 19.2                    | −34.08                 | 84.1                    |

series controls the presence of the high frequency variations (Nyquist limit). That means the geomagnetic field variations of time scales upto  $\sim 240$  min ( $2 \times 120$  min) exchange significant information between EEJ and Sq stations, while the variations of longer periodicities ( $>4$  h) do not exchange significant information. The observed response time delay could be due to the propagation time delay of the driver such as atmospheric gravity waves of periodicities between  $\sim 60$  and 240 min. The wave activity which is driving the variations at EEJ may take some time (few tens of minutes) to affect the variations at Sq. Therefore it is not fair to conclude that the variations in Sq are caused by the variations in EEJ. We only conclude from our analysis that there is a significant exchange of information between EEJ and Sq and hence these two systems are coupled, and the variations at EEJ appear before Sq station.

Further, it is noticed that the amplitude of TE (and also significant net flow) from TIR  $\rightarrow$  JAI is always higher than that from TIR  $\rightarrow$  NGP (Figs. 5b and d; 6 and 7), despite NGP being closer to TIR. We found similar observations when time series of different time resolutions (30 min to 2 h sampling interval in steps of 10 min) were examined reinforcing our findings from data with 1 h sampling interval. Whilst in the African sector, almost same amount of information flows from EEJ to both Sq stations. Though a robust result, this relatively less cross-talk between TIR and NGP is intriguing. This might be suggestive of an additional contribution exclusively present at NGP. Interestingly, based on the observations by earlier researchers, NGP is located close to the location of westward current, and the presence of the westward currents at NGP can reduce the TE values for TIR  $\rightarrow$  NGP.

In summary, the present study based on Indian and African sector clearly shows that the variations at EEJ and Sq are exchanging information between each other with a net flow of information from EEJ to Sq station for data of time resolutions between 30 and 120 min and hence are not independent systems. Therefore, present study reveals that EEJ and Sq are coupled systems.

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