Helicon mode driven by O^+ thermal anisotropy

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Abstract. Preliminary results from an investigation of the helicon instability in a plasma composed of protons, electrons and singly charged oxygen ions, are presented. The velocity distribution function for each plasma component is modelled by a bi-Lorentzian distribution, which allows each particle species to possess a power law tail of arbitrary spectral index. This permits us to model accurately the shape of the power law tails observed on particle species in the plasma sheet region, where the helicon mode is believed to play an important role. The presence of a hard power law tail on the oxygen component is found to dramatically enhance the maximum growth rate of the instability when the oxygen ions possess a small $T_\parallel > T_\perp$ anisotropy. Above a certain value of $T_\parallel/T_\perp$, however, this behaviour is reversed. The growth rate decreases as the spectral index of the protons is decreased. The relevance of these effects to the central plasma sheet region is briefly discussed.

I INTRODUCTION

Lakhina and Tsurutani [1,2] recently showed that oxygen ions (O^+) of ionospheric origin, possessing a $T_\parallel > T_\perp$ thermal anisotropy, can destabilise a low frequency, right hand circularly polarized helicon mode in the near Earth plasma sheet region. The helicon mode occurs at frequencies much lower than the proton gyrofrequency, usually the domain of the MHD modes in a proton-electron plasma. Despite its low frequency the helicon mode is non MHD in character, possessing a non-vanishing Hall current [1].

The most important sources of the magnetospheric O^+ component are the auroral region and the dayside polar cleft [3]. Ionospheric O^+ from these regions can feed the inner plasma sheet on typical substorm timescales. The helicon mode, which can be destabilised by such oxygen ions, is therefore important for our understanding of substorm phenomena. It has been suggested that helicon waves could lead to the fast current and flux penetration across the plasma sheet [4], thus affecting substorm dynamics. The relatively rapid characteristic growth time of the helicon mode instability, on timescales much shorter than the typical duration of enhanced convection events, likely means that the instability attains saturation during these

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events [1]. The low frequency electromagnetic turbulence produced by the helicon instability could scatter high energy electrons and help excite ion tearing modes that lead to substorm onset.

In the original papers by Lakhina and Tsurutani [1,2] the helicon mode was investigated in a plasma composed of bi-Maxwellian plasma components. There is, however, substantial evidence that charged particle species in the plasma sheet region possess high energy tails on their velocity distributions [5–8], and that these tails can be accurately fitted with a kappa distribution.

We investigate the helicon mode instability driven by O^+ thermal anisotropy in a model which allows each plasma component to possess a high energy power law tail with variable index κ, in addition to thermal anisotropy ($T_{\parallel} \neq T_{\perp}$). This model provides a more accurate picture of the measured particle distributions in the central plasma sheet (CPS) region, than does one based on bi-Maxwellian distributions. We have omitted particle drifts as we aim to isolate clearly the effects due to the high energy tails in this study. The results are compared and contrasted with those of Lakhina and Tsurutani [1].

II MODEL AND BASIC EQUATIONS

The kinetic model for particle species in the central plasma sheet is based upon the bi-Lorentzian velocity distribution function

$$f(v_\perp^2, v_z) = \pi^{-3/2} \frac{1}{\kappa^{3/2}\theta_\perp^2 \theta_\parallel} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-\frac{1}{2})} \left(1 + \frac{v_\perp^2}{\kappa\theta_\perp^2} + \frac{v_z^2}{\kappa\theta_\parallel^2}\right)^{-(\kappa+1)} \tag{1}$$

The distribution (1) has been normalized so that the integral over velocity space is unity. The constants $\theta_\perp$ and $\theta_\parallel$ are related to the perpendicular and parallel thermal speeds via $\theta_\perp = [(\kappa - \frac{3}{2})/\kappa]^{1/2}(2T_\perp/m)^{1/2}$ and $\theta_\parallel = [(\kappa - \frac{3}{2})/\kappa]^{1/2}(2T_\parallel/m)^{1/2}$. We shall call $\theta_\perp$ and $\theta_\parallel$ generalised thermal speeds. Distributions similar to (1) have recently been used for the electrons in a study [9] of the destabilization of the parallel whistler mode by electron thermal anisotropy in the electron foreshock.

Employing the distribution (1), the dispersion relation for the parallel propagating R-mode can be written [10,11]

$$\frac{k^2c^2}{\omega^2} = 1 + \sum_{\alpha=i,e,O^+} \frac{\omega_{p\alpha}^2}{\omega^2} \left\{ A_\alpha + \left[A_\alpha \left(\frac{\omega + \epsilon_\alpha \Omega_\alpha}{k\theta_{\parallel\alpha}} \right) + \frac{\omega}{k\theta_{\parallel\alpha}} \right] \right.$$
$$\left. \times \frac{(\kappa_\alpha - 1)^{3/2}}{\kappa_\alpha^{1/2}(\kappa_\alpha - \frac{3}{2})} Z_{\kappa_\alpha - 1}\left[\left(\frac{\kappa_\alpha - 1}{\kappa_\alpha} \right)^{1/2} \frac{\omega + \epsilon_\alpha \Omega_\alpha}{k\theta_{\parallel\alpha}} \right] \right\}, \tag{2}$$

where ω is the complex wave frequency, $k = |k_z|$, and for particle species α the plasma frequency is given by $\omega_{p\alpha} = (n_\alpha q_\alpha^2/\epsilon_0 m_\alpha)^{1/2}$, the gyrofrequency is given by $\Omega_\alpha = |q_\alpha \mathbf{B}_0|/m_\alpha$, charge sign is $\epsilon_\alpha = q_\alpha/|q_\alpha|$, the spectral index of the distribution is given by κ_α, and the temperature anisotropy is given by $A_\alpha = \theta_{\perp\alpha}^2/\theta_{\|\alpha}^2 - 1 = T_{\perp\alpha}/T_{\|\alpha} - 1$. Temperatures are measured in energy units throughout. The uniform magnetic field $\mathbf{B}_0$ is directed along the z axis of our coordinate system, i.e., $\mathbf{B}_0 = B_0\mathbf{e}_z$. The function $Z_\kappa(\zeta)$ appearing in (2) is the modified dispersion function, defined by Summers and Thorne [11] for positive integers $\kappa \geq 2$ and generalized to real positive values of $\kappa > \frac{3}{2}$ by Mace and Hellberg [12]. Equation (2) reduces to that used by Lakhina and Tsurutani [1] in the limit $\kappa \to \infty$ (since $\lim_{\kappa\to\infty} Z_\kappa(\zeta) = Z(\zeta)$, where Z is the plasma dispersion function of Fried and Conte [13]).

Due to the nature of the low frequency helicon mode and the extremely high β of the central plasma sheet it is difficult to approximate the above dispersion relation (2) to obtain analytical formulae for the real part of the frequency and the growth rate. In the following section we solve equation (2) without approximation.

III NUMERICAL RESULTS

We take as our nominal plasma sheet parameters the following: $n_i = 0.5\,\mathrm{cm}^{-3}$, $T_{\|e} = 0.5\,\mathrm{keV}$, $T_{\|p} = 5\,\mathrm{keV}$, $\beta_{\|O} = 3.5$, $B_0 = 10\,\mathrm{nT}$, $\kappa_e = 5.5$ and $\kappa_p = 5.5$. These parameters are based on those used by Lakhina and Tsurutani [1] in their study of the helicon instability. The values of κ_e and κ_p are representative of those occuring in the plasma sheet [7].

In the calculations we have kept the proton density constant at a value of $0.5\,\mathrm{cm}^{-3}$ and varied the density of O^+. This approach mimics the plasma dynamics in the CPS region where the ionospheric oxygen ions (and electrons, to maintain quasineutrality) periodically feed the region, supplementing the already quasineutral plasma there. It should be borne in mind, however, that the addition of oxygen ions increases the parallel electron beta $\beta_{\|e} = 2\mu_0 n_e T_{\|e}/B_0^2$ and the electron plasma frequency ω_{pe} as side effects.

Figure 1 illustrates the low frequency regime $0 < \omega < \Omega_p$ of the weakly unstable R-mode for a range of oxygen densities n_O/n_p. The oxygen temperature anisotropy $A_O = -0.1$, which represents a small $T_{\|O} > T_{\perp O}$ anisotropy, and $\kappa_O = 3.3$. The value of $\beta_{\|O} = 3.5$, which means that $\beta_{\|O} - \beta_{\perp O} = -A_O\beta_{\|O} = 0.35$. The instability in all cases is weakly growing at frequencies below the oxygen gyrofrequency Ω_O. The largest maximum growth rate and highest unstable frequency correspond to the smallest value of n_O/n_p.

Figure 2 illustrates the variation of the real frequency and growth rate as functions of A_O for a fixed oxygen ion density $n_O = 0.1n_p$. It is observed that the real frequency varies only slightly with A_O, whereas γ exhibits a marked dependence on this parameter. In particular, as A_O becomes increasingly negative, and hence $\beta_{\|O} - \beta_{\perp O}$ becomes larger, so the instability growth rate increases and the window of growing wavenumbers increases.

Figure 3 illustrates the variation of the real frequency and growth rate of the helicon mode as a function of the spectral index κ_O, for a small negative value of A_O, i.e., $A_O = -0.1$. The real frequency exhibits no dependence on κ_O to within

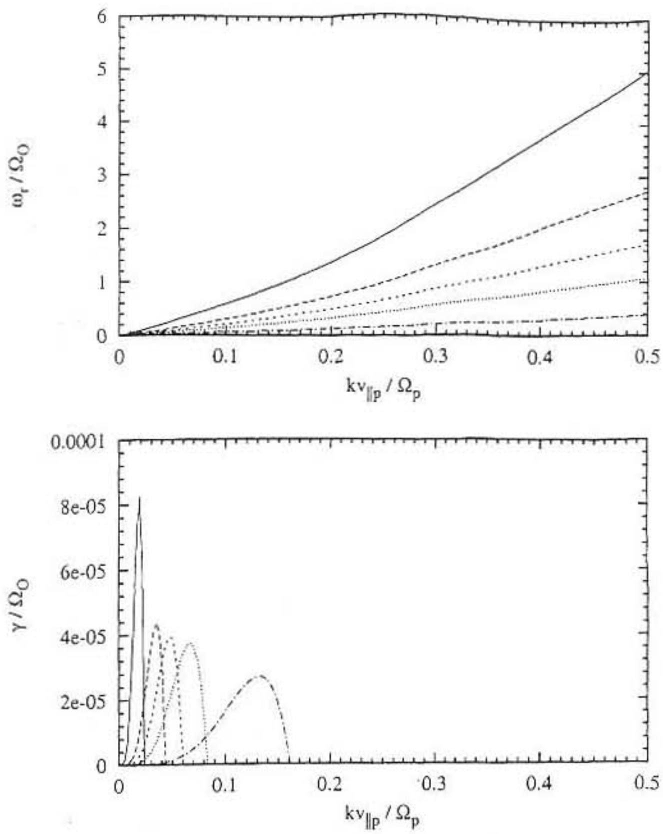

FIGURE 1. The low frequency region of the R-mode instability for a number of values of n_O/n_p: 0.1 (—), 0.5 (– –), 1 (- -), 2(···), 10(– · –). In all cases $A_O = -0.1$, $A_e = A_p = 0$, $\kappa_e = \kappa_p = 5.5$ and $\kappa_O = 3.3$. Here and in all subsequent figures, unlisted parameters correspond to those given in the text. The curves corresponding to the smallest values of n_O/n_p have the steepest slope.

graphical accuracy. The maximum growth rate, however, increases by well over three orders of magnitude as κ_O is decreased from 10 to 2, i.e., as the power-law tail on the O^+ distribution becomes ever more pronounced. Furthermore, the range of growing wavenumbers extends towards lower values of k as κ_O is decreased. The upper limit of the excited wavenumbers is independent of κ_O.

Figure 4 illustrates the effects of κ_O on the instability for a larger negative value of A_O. Here $A_O = -0.5$ and hence $\beta_{\|O} - \beta_{\perp O} = 1.75$, which is a very large β-anisotropy. In this case the effect of κ_O on the growth rate is not as dramatic as was found in figure 3. It is interesting to note that in this regime of larger thermal anisotropy, the effect of κ_O is precisely opposite to that observed in Figure 3, i.e., decreasing κ_O reduces the growth rate of the instability. In other words, a hard tail on the oxygen ions tends to diminish the instability in the regime $\beta_{\|O} - \beta_{\perp O} > 1$. However, common to both regimes of A_O is the fact that low values of κ_O excite the largest range of wavenumbers.

We have investigated the effects of varying the the spectral index of the proton

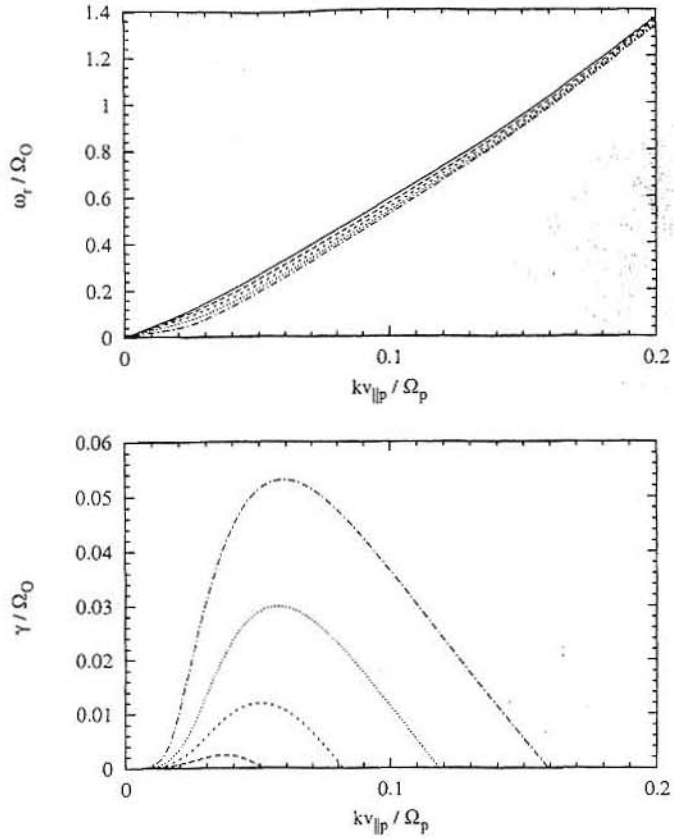

FIGURE 2. The low frequency region of the R-mode instability for a number of values A_O: -0.1 (—), -0.2 (– –), -0.3 (- -), -0.4 ($\cdots$) and -0.5 (– $\cdot\cdot$ –). In all cases $n_O = 0.1n_p$, $A_e = A_p = 0$ and $\kappa_e = \kappa_p = 5.5$, $\kappa_O = 3.3$.

velocity distribution on the dispersion characteristics and growth rate of the helicon instability (not shown). As the proton energy spectrum becomes harder, i.e., as κ_p decreases, so the growth rate of the instability rapidly decreases. However, above $\kappa_p \simeq 10$, when the particle statistics become quasi-Maxwellian, the maximum growth rate varies very slowly with κ_p. The dispersion characteristics of the helicon mode are very weakly dependent on this parameter.

We have investigated the effects of varying κ_e from κ_e = 3–10 for n_O/n_p = 0.5, $T_{\parallel p}/T_{\parallel e} = 10$, $\beta_{\parallel O} = 3.5$ and $A_O = -0.1$ (not shown). We find little or no dependence on κ_e for these parameters (to within graphical accuracy). Lakhina and Tsurutani [1] found furthermore that the instability growth rate is insensitive to the temperature anisotropy $T_{\parallel e}/T_{\perp e}$ of the electron distribution. Thus, it appears that the instability growth rate, and indeed the real part of the frequency, is insensitive to the exact details of the electron distribution.

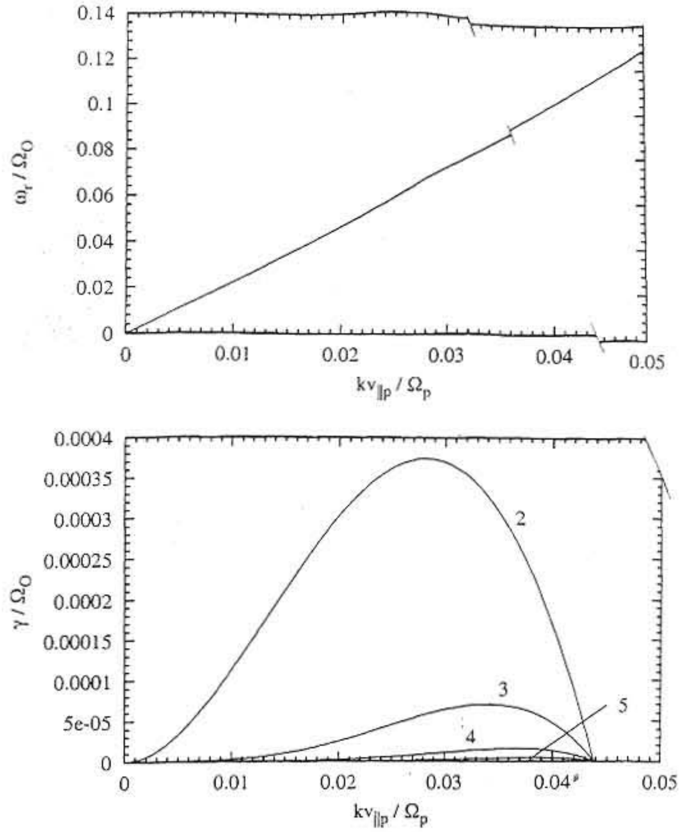

FIGURE 3. The low frequency region of the R-mode instability for a number of values of κ_O: 2, 3, 4, 5 and 10 (shown on curves). In all cases $n_O = 0.5 n_p$, $\kappa_e = \kappa_p = 5.5$, $A_O = -0.1$ and $A_e = A_p = 0$.

IV DISCUSSION AND CONCLUSIONS

We have investigated numerically the O^+ temperature anisotropy driven instability of the low frequency helicon mode in a plasma that comprises electrons, protons and O^+ ions, each having a power-law tail on their velocity distribution. The presence of power law tails on both the electron and proton species in the central plasma sheet region is now well known [5,7]. To account for the less certain details of the O^+ ions in the central plasma sheet and the variability of their abundance, we have varied their parameters over a correspondingly larger range of values.

This work extends the previous works of Lakhina and Tsurutani [1,2] in two important ways. First, as mentioned, we allow the particle distributions to possess a power law tail, which provides a more accurate statistical model of measured particle velocity distributions in the central plasma sheet. Second, we investigate a larger region of parameter space than was studied by Lakhina and Tsurutani.

The salient findings are as follows. The spectral index of the O^+ velocity distribution dramatically affects the growth rate of the helicon instability in the regime of small $T_{\parallel O}/T_{\perp O} - 1$, which we tentatively categorise as the regime where $-0.1 < A_O < 0$. In this regime the growth rate rapidly increases as the hardness

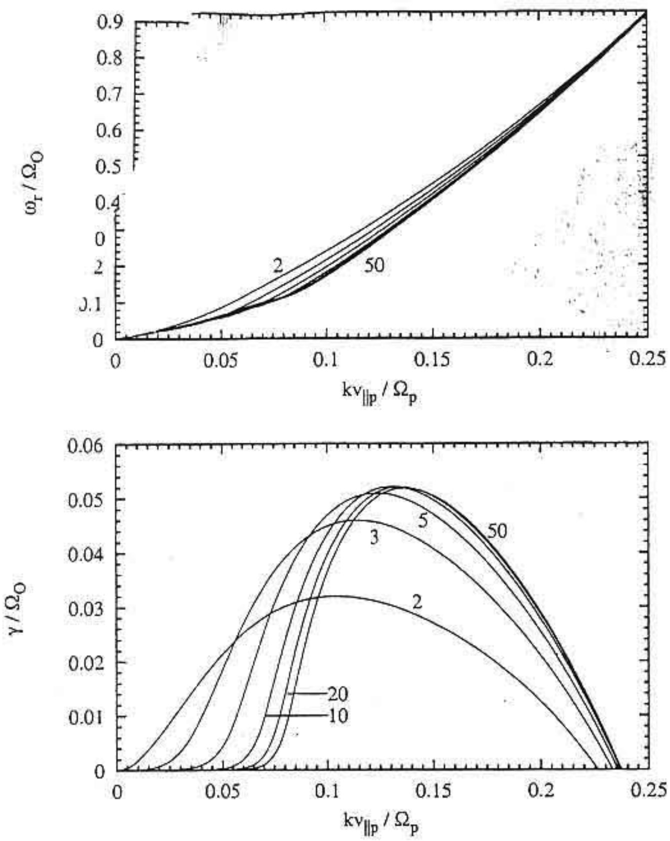

FIGURE 4. The low frequency region of the R-mode instability for a number of values of κ_O: 2, 3, 5 and 10 (shown on curves). In all cases $n_O = 0.5n_p$, $\kappa_e = \kappa_p = 5.5$, $A_O = -0.5$ and $A_e = A_p = 0$.

of the tail spectrum increases, i.e., as κ_O decreases. We observed an increase of over three orders of magnitude in the maximum growth rate of the instability as κ_O decreased from 10 to 2.

When the O^+ ions possess larger thermal anisotropy, i.e., when $T_{\parallel O}$ is sufficiently larger than $T_{\perp O}$, the growth rate is less sensitive to the value of κ_O. We have found that $A_O \sim -0.5 - -0.3$ characterises this regime. Here the dependence of the maximum growth rate on κ_O is precisely the opposite of that exhibited in the regime of small thermal anisotropy; a harder O^+ tail reduces the maximum growth rate of the helicon mode instability. The range of unstable wavenumbers and freqũencies is much larger than it is in the low thermal anisotropy regime, and the maximum growth rate is generally larger.

Since the protons [6] and electrons [7] in the central plasma sheet possess power law tails on their velocity distributions, their effects on the helicon instability are important. It is found that decreasing κ_p, i.e., increasing the hardness of the proton energy spectrum, reduces the growth rate of the helicon instability, whereas the instability is insensitive to variations in the spectral index of the electron distribution κ_e.

The exact nature of the helicon mode in the central plasma sheet will be determined by all of the above parameters, but primarily by the value of A_O, which plays the largest role in determining the maximum growth rate. If $T_{\parallel O}$ is sufficiently larger than $T_{\perp O}$, i.e., A_O is sufficiently negative ~ -0.5, then the maximum growth rate $\gamma_{\max}$ is much larger than it is for small (negative) values of $A_O \sim -0.1$. Accompanying this increased growth rate, the instability develops a more broadband nature, characterised by a larger window of growing wavenumbers. These two facts are generally true independently of the value of κ_O. Consequently, the nonlinearly saturated regime of the instability, that which is almost always observed in space plasmas, would be far more turbulent than the final weakly-turbulent state which would likely occur for small $|A_O|$. The large amplitude helicon turbulence so produced could be responsible for the twisting of the equilibrium magnetic field in the plasma sheet into flux ropes [1]. As discussed by Lakhina and Tsurutani [1], these properties of the helicon instability/turbulence give an indication that it might be playing a role in the processes related to oxygen ion bursts associated with multiple flux ropes in the distant magnetotail (and observed by GEOTAIL [14]).

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