Commun Nonlinear Sci Numer Simulat 22 (2015) 1322–1330

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Small amplitude electron acoustic solitary waves in a magnetized superthermal plasma

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ARTICLE INFO

Article history:
Received 18 October 2013
Received in revised form 25 July 2014
Accepted 28 July 2014
Available online 13 August 2014

Keywords:
Nonlinear waves
Space plasmas
Electron-acoustic solitons

ABSTRACT

The propagation of electron acoustic solitary waves in a magnetized plasma consisting of fluid cold electrons, electron beam and superthermal hot electrons (obeying kappa velocity distribution function) and ion is investigated in a small amplitude limit using reductive perturbation theory. The Korteweg–de-Vries–Zakharov–Kuznetsov (KdV–ZK) equation governing the dynamics of electron acoustic solitary waves is derived. The solution of the KdV–ZK equation predicts the existence of negative potential solitary structures. The new results are: (1) increase of either the beam speed or temperature of beam electrons tends to reduce both the amplitude and width of the electron acoustic solitons, (2) the inclusion of beam speed and temperature pushes the allowed Mach number regime upwards and (3) the soliton width maximizes at certain angle of propagation (α_m) and then decreases for $\alpha > \alpha_m$. In addition, increasing the superthermality of the hot electrons also results in reduction of soliton amplitude and width. For auroral plasma parameters observed by Viking, the obliquely propagating electron-acoustic solitary waves have electric field amplitudes in the range (7.8–45) mV/m and pulse widths (0.29–0.44) ms. The Fourier transform of these electron acoustic solitons would result in a broadband frequency spectra with peaks near 2.3–3.5 kHz, thus providing a possible explanation of the broadband electrostatic noise observed during the *Burst a*.

1. Introduction

Plasmas having superthermal electron and/or ion component are ubiquitous in space and astrophysical environments. Several observations have reported the presence of superthermal populations in the Earth's magnetosphere [15,6], solar wind [14], auroral region [2] etc., Such non-Maxwellian plasmas with high energy tail (referred as superthermal plasmas) are well modeled by kappa (κ) distribution functions. The kappa distribution function was first used by Vasyliunas [36] to fit the observations of OGO 1 and OGO 3 satellite's solar wind particle data. Later many authors have effectively used kappa velocity distribution function as a fit to real space observations [13,5,26]. Recently, Li and Cairns [19] studied the type III bursts and associated beams in coronal plasmas with kappa particle distributions. Their simulation results show that stronger electron acceleration is more easily observable in kappa plasmas than Maxwellian.

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http://dx.doi.org/10.1016/j.cnsns.2014.07.026
1007-5704/

Electron acoustic waves (EAWs) occur in several space plasma situation such as bow shock [3], magnetosheath [25], plasma sheet boundary layer [24], etc., They are associated with high frequency waves propagating at phase speed larger than the cold electron thermal velocities and less than the hot electron thermal velocities. Earlier studies on EAWs have used the Maxwellian or non-thermal velocity distribution function of the particles. Mace et al. [20] studied the arbitrary amplitude electron acoustic solitary waves in an unmagnetized plasma considering Boltzmann function for hot electron with fluid cool electron and ion to explain the electrostatic fluctuations, referred as cusp auroral hiss. Electron acoustic waves in an unmagnetized four-component plasma system consisting of two types of (cold and beam) electrons and ions have been studied by Lakhina et al. [18] to explain the spacecraft observations in the Earth's plasma sheet boundary layer. EAWs in an unmagnetized plasma with cold electrons and two different temperature isothermal ions obeying Boltzmann type distributions is studied by Kakad et al. [17] to explain the compressive and rarefactive bipolar electric fields of BEN type emission observed in the magnetospheric regions.

Singh and Lakhina [28] studied EAWs with nonthermal distribution of hot electrons, cold electrons and ions and presented a simple 1-D model. They have extended their study by including electron beam to investigate the effect of beam on electron acoustic solitary waves [29] and found both positive and negative potential/double layers. They have shown that, the addition of warm electron component is necessary to obtain positive potential structures.

The effects of magnetic field on obliquely propagating electron acoustic waves in two electron component magnetized plasma has been studied by Mamun et al. [23] using vortex like distribution for hot electrons in their model. EAWs also exist in a two component electron–ion magnetized plasmas provided that the ion temperature is much larger than the electron temperature. Such EAWs in a two component magnetized plasma is studied by Buti et al. [4]. Weakly nonlinear electron acoustic wave in a two electron component magnetized plasma is studied by Mace and Hellberg [21]. They have derived the Korteweg–de Vries–Zakharov–Kuznetsov (KdV–ZK) equation leading to plane and ellipsoidal soliton solutions to explain some of the two dimensional features of solitary wave observations.

Effect of magnetic field on nonthermal EAWs consisting of non-thermal hot electrons, cold electrons and stationary ions have been studied by Elwakil et al. [11]. They have added the fifth order dispersion term to the ZK equation to obtain higher order solution. It is found that the external magnetic field and obliquity alters the higher order amplitude of the electron acoustic solitons significantly.

Observations in space plasma region show the presence of electron beams associated with electrostatic solitary waves. The influence of beam on electron acoustic solitons with three electron populations consisting of Maxwellian electron beam in addition to the cold and hot population of electron has been studied by Singh et al. [30]. They found that parametric regime of electron acoustic waves strongly depend on the beam properties. Elwakil et al. [12] has studied both the relativistic effects and nonthermal effects of EAWs in a four component plasma with non thermal hot electron, relativistic cold electron, electron beam and stationary ions.

Electron acoustic waves featuring superthermal distribution for electrons and/or ions shows better approximations to the observations than Maxwellian or non-thermal distribution where high electric field amplitudes are observed. Devanandhan et al. [7,8] have studied the electron acoustic solitons in an unmagnetized plasma with two and three electron components and ions, respectively. In these studies, hot electrons are assumed to follow kappa distribution function. Occurrence of modulation instability (MI) in an unmagnetized plasma is investigated in the presence of superthermal electrons, cold electrons and ions [31]. Devanandhan et al. [9] studied the properties of the electron acoustic wave in a pure electron–ion magnetized plasma where ion temperature is much hotter than electron temperature. Arbitrary amplitude electron acoustic waves in a two electron component magnetized, cold plasma are studied by Sultana et al. [33] by assuming quasi neutrality condition. They have also derived the small amplitude limit by expanding the Sagdeev potential. Small amplitude weakly nonlinear electron acoustic waves are studied by Javidan and Pakzad [16] in a three component magnetized plasma using slightly different distribution function for the hot electrons from that of Sultana et al. [33]. Recently, the basic features of Zakharov–Kuznetsov (ZK) electrostatic solitons in a magnetized plasma consisting of warm ions and superthermal electrons is studied by Adnan et al. [1].

In the present paper, we study the electron acoustic solitary waves in a magnetized plasma consisting of hot electrons featuring kappa distribution, fluid cold electrons, an electron beam and ions. Here we extend the earlier work [33,16] by including an electron beam and finite temperature effects for all the species in the model. In Section 2, KdV–ZK equation is derived from basic equations and Section 3 describes the numerical results of solitary solutions for electron acoustic waves with kappa distributed hot electrons and an electron beam. Our results are summarized in Section 4.

2. Theoretical model

An infinite homogeneous, collisionless, magnetized plasma consisting of cold electron, cold electron beam, hot superthermal electron and ion is considered. The magnetic field $\mathbf{B_0}$ is assumed to lie along the z-axis. We consider the hot superthermal electrons to follow the κ distribution function as given by Thorne and Summers [35],

$$f_h(v) = \frac{N_{h0}}{\pi^{3/2}\theta^3} \frac{\Gamma(\kappa)}{\sqrt{\kappa}\Gamma\left(\kappa - \frac{1}{2}\right)} \left(1 + \frac{v^2}{\kappa\theta^2}\right)^{-(\kappa+1)} \tag{1}$$

where $\theta^2 = (2 - 3/\kappa)T_h/m_e$ is the modified thermal speed, κ is the spectral index $(\kappa > 3/2)$ and $\Gamma(\kappa)$ is the gamma function. The κ distribution approaches to the Maxwellian in the limit $\kappa \to \infty$. Hot electron density which is obtained by replacing v^2/θ^2 in the distribution function given by Eq. (1) with $(v^2/\theta^2) - (2e\Phi/m_e\theta^2)$ and integrating it over the velocity space can be written in the normalized form as,

$$n_h = \left[1 + \frac{\phi}{\kappa - 3/2}\right]^{-\kappa+1/2} \tag{2}$$

The basic set of fluid equations governing the dynamics of the cool electrons, electron beam and ions can be written as

$$\frac{\partial n_j}{\partial t} + \nabla(n_j v_j) = 0 \tag{3}$$

$$\mu_j n_j \left[\frac{\partial v_j}{\partial t} + (v_j.\nabla) v_j\right] = Z_j n_j \nabla\phi - \nabla P_j - Z_j \mu_j n_j \Omega_j (v_j \times z) \tag{4}$$

$$\frac{\partial P_j}{\partial t} + v_j.\nabla P_j + \gamma P_j \nabla . v_j = 0 \tag{5}$$

$$\nabla^2\phi = n_c + n_h + n_b - n_i \tag{6}$$

In our analysis, we have taken arbitrary adiabatic index defined as $\gamma = \frac{2+N}{N}$; N is the number of degrees of freedom. Here the subscript j denotes cold electrons (c), electron beam (b) and ion fluid (i) of the system. Here $\mu_j = m_j/m_e$ is the mass ratio, $Z_j = \pm 1$ for electrons and ions, respectively, and $\Omega_j = \omega_{cj}/\omega_{pe}$ where $\omega_{cj} = eB_0/m_j c$ is the cyclotron frequency of the jth species and $\omega_{pe} = (4\pi N_0 e^2/m_e)^{1/2}$ is the total electron plasma frequency.

In the above equations we have used the following normalizations: lengths (x,z) by Debye length $(T_h/4\pi N_0 e^2)^{1/2}$, velocities (v) by thermal velocity of hot electrons $(T_h/m_e)^{1/2}$, time (t) by inverse of plasma frequency ω_{pe}^{-1}, electrostatic potential (Φ) by (T_h/e), densities by equilibrium density $N_0 = N_{h0} + N_{b0} + N_{c0} = N_{i0}$ and pressure (P) by $(N_0 T_h)$.

We use the reductive perturbation technique [27,34] to derive the small amplitude soliton solution from the Eqs. (2)–(6). According to this method, we introduce the following stretching

$$\xi = \epsilon^{1/2}x, \quad \eta = \epsilon^{1/2}y, \quad \theta = \epsilon^{1/2}(z - Vt), \quad \tau = \epsilon^{3/2}t \tag{7}$$

and expansions

$$\begin{aligned}
n_j &= n_{j0} + \epsilon n_{j1} + \epsilon^2 n_{j2} + \epsilon^3 n_{j3} + \cdots \\
P_j &= P_{j0} + \epsilon P_{j1} + \epsilon^2 P_{j2} + \epsilon^3 P_{j3} + \cdots \\
\phi &= \epsilon\phi_1 + \epsilon^2\phi_2 + \epsilon^3\phi_3 + \cdots \\
v_{jx} &= \epsilon^{3/2} v_{j1x} + \epsilon^2 v_{j2x} + \epsilon^{5/2} v_{j3x} + \cdots \\
v_{jy} &= \epsilon^{3/2} v_{j1y} + \epsilon^2 v_{j2y} + \epsilon^{5/2} v_{j3y} + \cdots \\
v_{jz} &= v_{j0} + \epsilon v_{j1z} + \epsilon^2 v_{j2z} + \epsilon^3 v_{j3z} + \cdots
\end{aligned} \tag{8}$$

where ϵ represents the smallness parameter and V is the phase speed. Substituting Eqs. (7) and (8) in Eqs. (2)–(6) and equating the coefficients of lowest order (ϵ) of the continuity and momentum equations, we obtain the following set of linear equations

$$n_{h1} = n_{h0}\left(\frac{2\kappa - 1}{2\kappa - 3}\right)\phi_1 \tag{9}$$

$$-(V - v_{j0})\frac{\partial n_{j1}}{\partial\theta} + n_{j0}\frac{\partial v_{j1z}}{\partial\theta} = 0 \tag{10}$$

$$Z_j n_{j0}\frac{\partial\phi_1}{\partial\xi} - \frac{\partial P_{j1}}{\partial\xi} - Z_j\mu_j\Omega_j n_{j0} v_{j1y} = 0 \tag{11}$$

$$Z_j n_{j0}\frac{\partial\phi_1}{\partial\eta} - \frac{\partial P_{j1}}{\partial\eta} + Z_j\mu_j\Omega_j n_{j0} v_{j1x} = 0 \tag{12}$$

$$-\mu_j n_{j0}(V - v_{j0})\frac{\partial v_{j1z}}{\partial\theta} = Z_j n_{j0}\frac{\partial\phi_1}{\partial\theta} - \frac{\partial P_{j1}}{\partial\theta} \tag{13}$$

$$-(V - v_{j0})\frac{\partial P_{j1}}{\partial\theta} + \gamma P_{j0}\frac{\partial v_{j1z}}{\partial\theta} = 0 \tag{14}$$

$$n_{c1} + n_{h1} + n_{b1} = n_{i1} \tag{15}$$

Substituting the number densities n_{c1}, n_{h1}, n_{b1} and n_{i1} obtained from Eqs. (9)–(14) in Eq. (15), we obtain the following dispersion relation

$$n_{h0}\left(\frac{2\kappa-1}{2\kappa-3}\right) - \frac{n_{c0}}{V^2 - \gamma T_c} - \frac{n_{b0}}{(V - v_{b0})^2 - \gamma T_b} - \frac{1}{\mu_i V^2 - \gamma T_i} = 0 \tag{16}$$

The dispersion relation does not depend directly on the magnetic field and it is similar to the dispersion relation for electron-acoustic waves as in the case of unmagnetized plasmas (cf. Eq. (7) of [8]). The effects of magnetic fields appear in higher order terms because of the expansions used in Eq. (8) (see Eq. (28) below). It must be pointed out here that in the three component, cold plasma limit (neglecting the electron beam and finite temperatures of the species and considering ions as stationary) our dispersion relation (16) reduces to Sultana et al. [33] in a weakly magnetized plasma (refer to their Eq. (9)). That is $V = \sqrt{\frac{n_{c0}}{n_{h0}}}\sqrt{\frac{2\kappa-3}{2\kappa-1}} = \sqrt{\beta/c_\kappa}$ (in Sultana et al. [33] notations $\beta = \frac{n_{c0}}{n_{h0}}$ and $c_\kappa = \frac{2\kappa-1}{2\kappa-3}$). Further, in the absence of cold and beam electron components, the dispersion relation given by Eq. (16) reduces to the one given by Adnan et al. [1] (cf. Eq. (25)) for ion-acoustic solitons in magnetized plasma with anisotropic ion pressure and kappa distributed electrons.

Next, the coupled set of equations is obtained by grouping the terms of higher order of ϵ from superthermal hot electron density, continuity, momentum, equation of state and Poisson's equations and is given by

$$n_{h2} = n_{h0}\left[\left(\frac{2\kappa-1}{2\kappa-3}\right)\phi_2 + \frac{(2\kappa-1)(2\kappa+1)}{2(2\kappa-3)^2}\phi_1^2\right] \tag{17}$$

$$\frac{\partial n_{j1}}{\partial \tau} - (V - v_{j0})\frac{\partial n_{j2}}{\partial \theta} + n_{j0}\frac{\partial v_{j2x}}{\partial \xi} + n_{j0}\frac{\partial v_{j2y}}{\partial \eta} + n_{j0}\frac{\partial v_{j2z}}{\partial \theta} + n_{j1}\frac{\partial v_{j1z}}{\partial \theta} + v_{j1z}\frac{\partial n_{j1}}{\partial \theta} = 0 \tag{18}$$

$$(V - v_{j0})\frac{\partial v_{j1x}}{\partial \theta} = Z_j\Omega_j v_{j2y} \tag{19}$$

$$(V - v_{j0})\frac{\partial v_{j1y}}{\partial \theta} = -Z_j\Omega_j v_{j2x} \tag{20}$$

$$\mu_j n_{j0}\frac{\partial v_{j1z}}{\partial \tau} - \mu_j n_{j0}(V - v_{j0})\frac{\partial v_{j2z}}{\partial \theta} - \mu_j n_{j1}(V - v_{j0})\frac{\partial v_{j1z}}{\partial \theta} + \mu_j n_{j0} v_{j1z}\frac{\partial v_{j1z}}{\partial \theta} = Z_j n_{j0}\frac{\partial \phi_2}{\partial \theta} + Z_j n_{j1}\frac{\partial \phi_1}{\partial \theta} - \frac{\partial P_{j2}}{\partial \theta} \tag{21}$$

$$\frac{\partial P_{j11}}{\partial \tau} - (V - v_{j0})\frac{\partial P_{j2}}{\partial \theta} + v_{j1z}\frac{\partial P_{j1}}{\partial \theta} + \gamma P_{j1}\frac{\partial v_{j1z}}{\partial \theta} + \gamma P_{j0}\left(\frac{\partial v_{j2x}}{\partial \xi} + \frac{\partial v_{j2y}}{\partial \eta} + \frac{\partial v_{j2z}}{\partial \theta}\right) = 0 \tag{22}$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2} + \frac{\partial^2 \phi_1}{\partial \theta^2} = n_{c2} + n_{h2} + n_{b2} - n_{i2} \tag{23}$$

In order to obtain the KdV–ZK equation, following procedure is adopted. All the second order terms in Eqs. (17)–(23) are eliminated using the Eqs. (9)–(14) along with the dispersion relation (16). After some algebraic manipulation and regrouping the similar terms, we obtain the following KdV–ZK equation for the propagation of electron-acoustic solitons in a magnetized four-component plasma with kappa distribution of electrons

$$\frac{\partial \phi_1}{\partial \tau} + a\phi_1\frac{\partial \phi_1}{\partial \theta} + b\frac{\partial^3 \phi_1}{\partial \theta^3} + c\frac{\partial}{\partial \theta}\left(\frac{\partial^2 \phi_1}{\partial \xi^2} + \frac{\partial^2 \phi_1}{\partial \eta^2}\right) = 0 \tag{24}$$

where the coefficients a, b and c are given by

$$a = \frac{B}{A}; \quad b = \frac{1}{A}; \quad c = \frac{C}{A}; \tag{25}$$

$$A = \Sigma_j \frac{2Z_j^2 n_{j0}\mu_j(V - v_{j0})}{\left[\mu_j(V - v_{j0})^2 - \gamma T_j\right]^2} \tag{26}$$

$$B = -2n_{h0}\frac{(2\kappa-1)(2\kappa+1)}{2(2\kappa-3)^2} - \Sigma_j \frac{\gamma Z_j^3 n_{j0}\left[\mu_j(V - v_{j0})^2 + T_j\right]}{\left[\mu_j(V - v_{j0})^2 - \gamma T_j\right]^3} \tag{27}$$

$$C = 1 + \Sigma_j \frac{n_{j0}\mu_j(V - v_{j0})^4}{\Omega_j^2\left[\mu_j(V - v_{j0})^2 - \gamma T_j\right]^2} \tag{28}$$

Eq. (24) is derived for an arbitrary 3D perturbation. To solve Eq. (24), we assume waves to be propagating in the x–z plane, therefore assume $\frac{\partial}{\partial \eta} = 0$ and introduce the following transformation $Z = \theta\cos\alpha + \xi\sin\alpha - U\tau$ with boundary condition that ϕ_1 and $d\phi_1/dZ \to 0$ as $Z \to \infty$. Thus, the plane wave solution of Eq. (24) for 2D perturbation is given as,

$$\phi_1 = \phi_0 sech^2\left(\frac{Z}{\Lambda}\right) \tag{29}$$

where the amplitude ϕ_0 and width Λ of the electron acoustic solitons are given as

$$\phi_0 = \frac{3U}{a\cos\alpha}, \quad \Lambda = 2\left[\frac{\cos\alpha\left(b\cos^2\alpha + c\sin^2\alpha\right)}{U}\right]^{1/2} \tag{30}$$

where U is the speed of the soliton relative to the phase speed, V, and α is the angle between the direction of wave propagation and external magnetic field $\mathbf{B_0}$. From the Eq. (29) we can investigate the parametric regime of the solitary waves of our model. For that, we find the root of the dispersion relation given by Eq. (16) which is of six degrees in V. Eq. (16) can be reduced to a bi-quadratic equation and it can be solved analytically if we neglect the ion dynamics. Since we retain ion dynamics, we have numerically solved the complete Eq. (16) for all six roots by employing Muller method. Among the six roots, three roots are negative and three roots are positive. We consider only the positive roots as the negative roots imply the waves propagating in opposite direction. The two positive roots are of the order of 10^{-3} and 10^{-2} which do not fall into the electron-acoustic regime. We have taken the third positive real root which is of order of unity and falls in electron-acoustic range.

Before we move on to the numerical results, it is important to mention here that by dropping the electron beam component and temperature effects, we obtain the usual two electron component electron acoustic waves in magnetized plasmas. Further, in the absence of cold and beam electron components, our expressions for coefficients a, b and c matches with the coefficients A, B and C, respectively of [1] (cf. Eq. (25)) (in the limit of isotropic pressure, i.e., $p_1 = p_2$ (cf. Eq. (33)). In reference [33], Sultana et al. have studied electron-acoustic solitary waves in three-component plasma consisting of cold electrons (having zero temperature) and hot (having kappa-distribution) electrons and immobile ions. They have employed the quasi-neutrality condition to arrive at the energy integral. In the present paper, we have extended their work by including finite temperature for cold electrons, an intermediate temperature electron beam and dynamical ions having finite temperatures. In the absence of electron beam, cold electrons (zero temperature) and immobile ions, the maximum soliton amplitude (ϕ_0) given by Eq. (30) reduces to the maximum amplitude (ϕ_m) given by Eq. (20) in Sultana et al. [33] and can be written in simplified form (in our notations) as

$$\phi_0 = \frac{6U}{\cos\alpha}\left[\frac{n_{c0}}{n_{h0}}\left(\frac{2\kappa - 3}{2\kappa - 1}\right)\right]^{1/2}\left\{\frac{n_{c0}}{n_{h0}}\left(\frac{2\kappa + 1}{2\kappa - 1}\right) + 3\right\}^{-1} \tag{31}$$

Since, Sultana et al. [33] have not provided the explicit expressions for the coefficients a, b and c, therefore, these could not be compared with theirs. Further, the maximum soliton amplitude is also similar to the one obtained by Javidan and Pakzad [16] (cf Eq. (8)). However, they use slightly different kappa-distribution for hot electrons.

It can also be seen from the Eq. (30) that the maximum width of the soliton (ie. Λ_{max}) is found at an angle $\alpha_m = tan^{-1}[\sqrt{(2c - 3b)/c}]$. For an angle of propagation $0^o < \alpha < \alpha_m$, the width of the solitons is bounded by $\sqrt{4b/U} \leqslant \Lambda \leqslant \Lambda_{max}$. For an angle $\alpha > \alpha_m$, the soliton width decreases with increasing α and tends to zero as $\alpha \to 90^o$. The behavior of solitary width for various values of κ is plotted in Fig. 1(a). The amplitude of the solitons given by Eq. (29) can be restricted by choosing a value of U in such a way that the assumption of small amplitude theory remains valid, i.e., for all values of $U \ll 1$.

3. Numerical results and discussion

For numerical computations, the parameters from the Viking observations in the dayside auroral region [10] have been taken to study the effect of superthermality, beam temperature, beam velocity and the obliquity on the properties of electron acoustic solitons. The chosen parameters from the observation of *Burst a* are: cold electron density $n_{c0} = 0.5\ \mathrm{cm}^{-3}$, hot electron density $n_{h0} = 2.0\ \mathrm{cm}^{-3}$, beam electron density $n_{b0} = 1.0\ \mathrm{cm}^{-3}$, hot electron temperature $T_h = 250$ eV and ambient magnetic field, $B_0 = 3570$ nT. Eq. (29) is solved numerically to obtain the soliton solutions.

For all numerical computations, we have taken $\gamma = 2$ which is valid for a system with 2 degrees of freedom. Fig. 1(a) and (b) shows the variation of width and amplitude of the solitons, respectively with different values of κ. Other normalized parameters used in the figures are as follows: $\alpha = 30^0$, $U = 0.05$, $v_{b0} = 0.1$, $\Omega_{c,b} = 6.78$ and $\Omega_i = 3.68 \times 10^{-3}$. It can be seen from the Fig. 1(a) that the width of the soliton increases with angle of propagation α and reaches a maximum value,

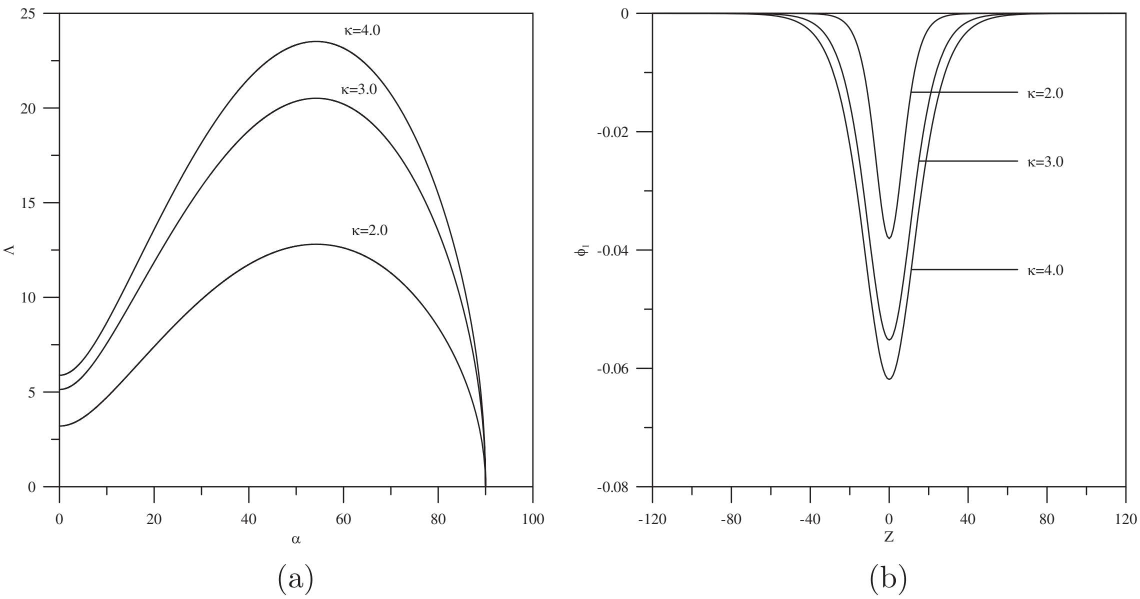

Fig. 1. (a) Variation of width and (b) soliton amplitude for various values of κ. Here, the curves are shown for κ = 2 (V = 0.589681), 3 (V = 0.75449) and 4 (V = 0.814161). Other values are $n_{b0} = 1.0\ \mathrm{cm}^{-3}$, $n_{h0} = 1.5\ \mathrm{cm}^{-3}$, $n_{c0} = 0.2\ \mathrm{cm}^{-3}$, $T_c = T_i = 0.001$, $T_b = 0.01$, $B_0 = 3570$ nT, $\alpha = 30^0$, U = 0.05, $v_{b0} = 0.1$, $\Omega_{c,b} = 6.78$, $\Omega_i = 3.69 \times 10^{-3}$.

Λ_{max} at $\alpha_m = 54.2^o$ (in this case) and then decreases for $\alpha > \alpha_m$. The behavior of width of the superthermal solitons is identical to the behavior of non-thermal plasmas reported by Mamun et al. [22].

Fig. 1(b) shows that the increase in superthermality (i.e. lower values of kappa) leads to decrease in the solitary potential amplitude which appears is in conformity with the results for small amplitude electron acoustic solitons obtained by Sultana et al. [33] and Javidan and Pakzad [16]. However, this result appears to be contradicting the results for the case of unmagnetized plasmas in which higher superthermality accounts for the higher solitary amplitudes for a fixed Mach number [31,9]. Here, it has to be clearly mentioned that the value of the Mach number V (or root of the polynomial equation arising from dispersion relation (16)) is different for different kappa values. Thus, the Fig. 1(b) effectively describes the variation of soliton amplitude with the Mach number through changes in kappa values. Therefore, the parameter kappa explaining the intense solitary waves for a fixed speed M in arbitrary amplitude case differs from this case. The maximum normalized amplitude

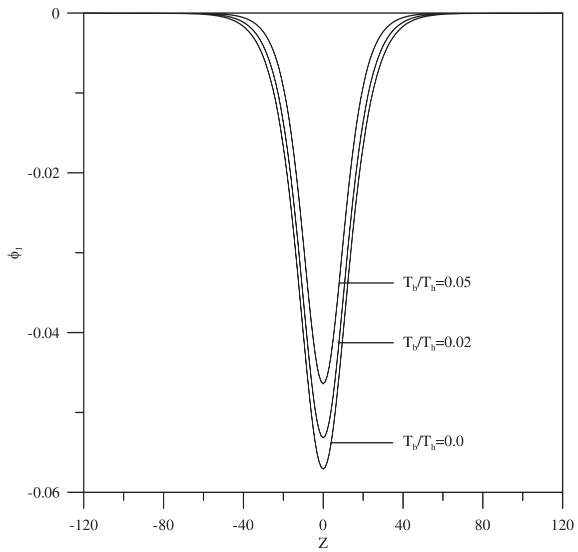

Fig. 2. Variation of soliton amplitude for various values of T_b/T_h for $\kappa = 3.0$. Here, the curves are shown for T_b/T_h = 0.0 (V = 0.742781), 0.02 (V = 0.766228) and 0.05 (V = 0.801448). Other values are same as Fig. 1.

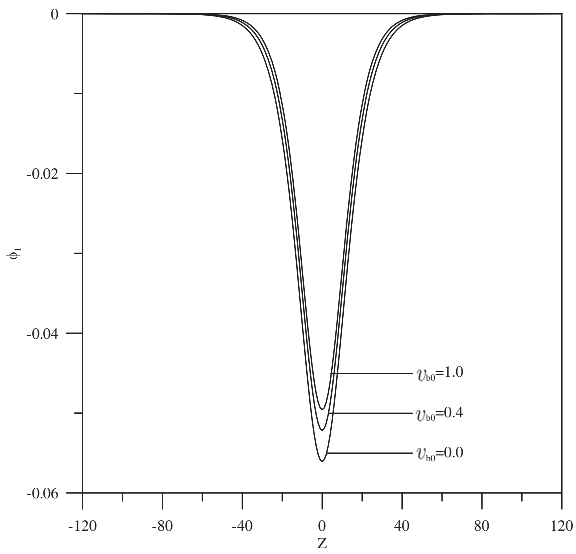

Fig. 3. Variation of soliton amplitude for various values of v_{b0} for $\kappa = 3.0$. Here, the curves are shown for v_{b0} = 0.0 (V = 0.681666), 0.5 (V = 1.00996) and 1.0 (V = 1.58241). Other values are same as Fig. 1.

ϕ_{max} is found to be (0.038–0.061) for κ = 2.0–4.0. The corresponding electric field amplitude for the parameters of Fig. 1(b) gives us higher amplitude for κ distribution of electrons than the Maxwellian distribution (i.e. similar to Fig. 4(b) of [32]).

In Fig. 2 the effect of beam electron temperature is shown for fixed value of nonthermal parameter, κ = 3.0. Other parameters used are same as in Fig. 1. The electrostatic soliton amplitude is maximum (ϕ_{max} = 0.057) for T_b/T_h = 0 case. The inclusion of beam temperature effects reduces the solitary potential amplitudes significantly (for T_b/T_h = 0.05 case we obtain ϕ_{max} = 0.046). Further, the width also narrows down with the increase in beam electron temperature. It is also seen numerically that as the beam temperature increases the phase speed for which soliton solutions are obtained also increases. In Fig. 3, variation of soliton amplitude for various values of beam speed for $\kappa = 3$ is shown. It is obvious from Fig. 3 that the increase in beam electron speed reduces the solitary wave amplitudes and narrows down the width of the solitons. For electron beam speed v_{bo} = (0.0–1.0) the normalized maximum soliton amplitude ϕ_{max} is found to be in the range (0.056–0.049). In this case also the phase speed increases with the increase in beam speed. It is further noted that the range

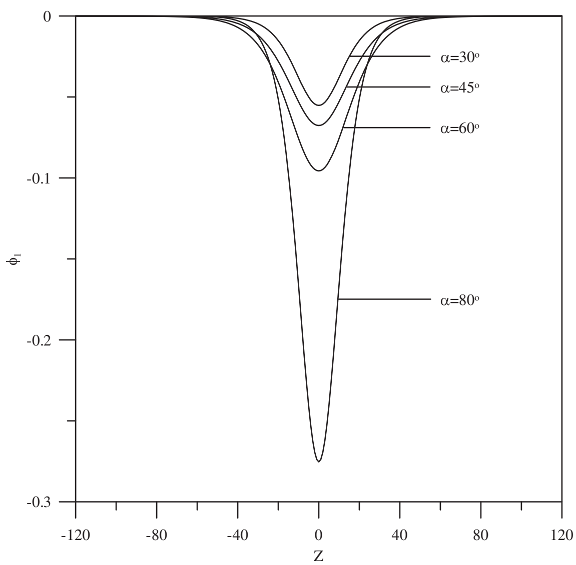

Fig. 4. Variation of soliton amplitude for various values of α for $\kappa = 3.0$. Here, the curves are shown for α = 30°, 45°, 60° and 80° (V = 0.75449) Other values are same as Fig. 1.

of phase speeds for which soliton solutions have been obtained are narrower in the previous case as compared to the later case.

Variation of solitary potential amplitude with the angle of propagation, "α" is plotted in Fig. 4. It is seen that the soliton amplitude increases for increase in angle of propagation, α. We obtained the maximum amplitude ϕ_{max} = (0.055–0.27) for α = 30°–80°. However, the soliton width increases for increase in angle of propagation and decreases beyond certain limit.

4. Conclusions

The electron acoustic solitons have been studied in a four component, weakly magnetized plasma consisting of superthermal hot electrons having kappa distribution, fluid cold electrons, electron beam and ions. KdV–ZK equation is derived from multi-fluid equations to analyze the electron acoustic solitons in a small amplitude region. It is found that only negative potential solitary structures exist for the parameters considered here. Theoretical results are applied to the Viking observations [10] to explain the broadband electrostatic noise (BEN) observations in terms of electron acoustic solitons. In the Figures we have directly used the observed parameters in normalized form and later, calculated the electric field amplitudes from our theoretical model.

Our results show that lower values of kappa gives higher electric field amplitudes and an increase in beam temperature decreases the electric field amplitude. The detailed analysis of soliton width shows that it maximizes at an angle of propagation ($\alpha_m = 54.2^o$), and decreases beyond $\alpha > \alpha_m$. The new findings from this study show that both the amplitude and width of the electron acoustic solitons decreases with the increase of either the electron beam speed or temperature and it also pushes the Mach number upwards for which soliton solutions are obtained. Further, at certain angle of propagation, i.e., $\alpha = \alpha_m$, the soliton width maximizes and then decreases for $\alpha > \alpha_m$. The unnormalized electric field amplitudes calculated for various angles of propagation, $\alpha = 30^\circ - 80^\circ$, are found to be in the range (7.8–45) mV/m.. This shows that electron acoustic solitons propagating at large angle to B_0 have higher electric field amplitudes. The corresponding spatial width and pulse width are found to be in the range of (1500–2260) m and (0.29–0.44) ms, respectively whereas the solitary wave speed is found to be about 5100 km/s. The Fourier transform of these solitary structures would produce broadband electrostatic noise with peaks near the inverse pulse width [32], i.e. between 2.3 and 3.5 kHz. The BEN spectra for the *Burst a* [10] peaks near 3 to 4 kHz and has maximum electric field 37 mV/m. Thus our results are in good agreement with the observations.

The limitation of the model is that it can explain solitary potential structures in weakly magnetized superthermal plasmas as the effect of magnetic field comes through the second order terms. This can be further improved by considering higher order terms. In space plasma situations, where high magnetic field environment is present, this model may not fit as such. In these situations one has to use an expansion scheme different from Eq. (8) for the perpendicular velocity components so that the magnetic field effects are felt even at the first order of ϵ, i.e., the dispersion relation contains the terms that depend on the magnetic field.

Acknowledgments

S.V.S. and R.B. would like to thank Department of Science and Technology, New Delhi, India and NRF South Africa, respectively for the financial support. The work was done under Indo-South Africa Bilateral Project "Linear and nonlinear studies of fluctuation phenomena in space and astrophysical plasmas". G.S.L. thanks the National Academy of Sciences, India for the support under the NASI-Senior Scientist Platinum Jubilee Fellowship.

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