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The Effect of Dust Grain Temperature and Dust Streaming on Electrostatic Solitary Structures in a Non-Thermal Plasma

S. K. Maharaj[1], S. R. Pillay[2], R. Bharuthram[2], S. V. Singh[3] and G. S. Lakhina[3]

[1]Hermanus Magnetic Observatory, Hermanus, 7200, South Africa
[2]University of KwaZulu-Natal, Durban, 4000, South Africa
[3]Indian Institute of Geomagnetism, Plot No. 5, Sector-18, New Panvel (W), Navi Mumbai, 410218, India

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Abstract

The influence of dust temperature and dust streaming on dust acoustic solitary waves is investigated in an unmagnetized dusty plasma comprising negatively charged dust, non-thermal ions and Boltzmann electrons. It is found that the conditions which support the coexistence of both compressive and rarefactive solitons is altered by the dust temperature and by dust streaming.

1. Introduction

The study of plasmas with massive negatively charged dust particles has become the focus of research in the last decade due to the widespread occurence of such plasmas in space and astrophysical environments.

Dusty plasmas are responsible for the introduction of a new ultralow frequency domain of wave modes and instabilities, the most popular of these being the dust acoustic wave (DAW) first reported by Rao *et al.* [1]. The existence of this wave has been verified in numerous laboratory experiments [2–4].

It is well known that the velocity distribution functions of the different charged particle species that make up a plasma, play a crucial role in influencing the linear as well as nonlinear properties of the plasma. The Maxwellian distribution function, which is in thermal equilibrium, is most frequently used in collisionless plasmas. However various observations of fast ions and electrons in space environments indicate that these particles have velocity distributions that are not in thermal equilibrium. For example, non-thermal ions have been observed in the Earths bow-shock [5]. The loss of energetic ions from the upper ionosphere of Mars has been observed through satellite observations [6]. More recently, fast protons have been observed near the Moon [7]. Non-thermal ion populations which could be modelled by a Kappa distribution have been found to occur in the magnetospheres of planets like Jupiter and Saturn [8, 9]. The above observations justify the need to study the influence of non-thermal velocity distributions on waves and instabilities in space plasmas.

Many authors have adopted non-thermal velocity distributions in nonlinear plasma studies. Cairns *et al.* [10] proposed that the non-thermal nature of electrons in a two component model comprising Boltzmann ions as well, could support the simultaneous existence of solitary waves with both positive and negative potential which could possibly account for the observations made by the Freja satellite. They found that when they considered a thermal plasma, only a single solitary wave was supported. Mamun *et al.* [11] employed a model comprising non-thermal ions and negatively charged dust and reported that solitary waves with positive and negative potential can coexist. If we compare with an earlier work by Mamun *et al.* [12] employing a model comprising Boltzmann ions and negatively charged dust particles, it was found that for this case, only solitary waves with negative potential exist. Gill and Kaur [13] studied nonlinear dust acoustic waves in a two component plasma comprising non-thermal ions and a negatively charged dust component. They incorporated the effects of dust temperature into the model and found that dust temperature can influence whether rarefactive solitons can simultaneously coexist with compressive solitons in the non-thermal plasma. Mendoza-Briceno *et al.* [14] employed a two component model of non-thermal ions and negatively charged dust and used the pseudopotential method to determine, theoretically, the dependence of the required cutoff values of the non-thermal parameter α and Mach number M on dust temperature for compressive and rarefactive solitons to simultaneously exist. The relative streaming between charged particle environments also occurs in many space and astrophysical environments. Mahmood and Saleem [15] used a dust-ion and dust-ion-electron model where the electrons and ions are Boltzmann distributed. Dust thermal effects were not considered ($T_d = 0$). They found that the amplitude of the dust acoustic solitary structures decreases significantly with an increase in the dust streaming velocity. It was also found that the width of the solitary structure is narrower when free electrons are present in the dusty plasma. This justifies the need for a study which includes electrons in a dust-ion plasma.

The present work extends the two component dust-ion model of Mendoza-Briceno *et al.* [14] to include electrons. We adopt a three-component model including electrons in our study because it is mentioned in [16] that the complete depletion of electrons (being attached on the dust grains) is not always possible. The effects of dust streaming is also considered in this work.

2. Model and governing equations

We consider a three component unmagnetized dusty plasma comprising a negatively charged drifting dust component, stationary non-thermal ions and Boltzmann electrons. In our analysis, we closely follow the approach of Mendoza-Briceno *et al.* in ref. [14].

For overall charge neutrality at equilibrium, we require

$$n_{i0} = n_{e0} + Z_d n_{d0} \tag{1}$$

where n_{i0}, n_{e0} and n_{d0} are the equilibrium number densities of the ions, electrons and dust particles respectively. Z_d is the number of electrons that reside on a single dust grain.

For the low phase velocity dust acoustic oscillations, the dust dynamics are governed by the continuity, momentum and pressure balance equations, respectively

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \mathbf{v_d}) = 0, \tag{2}$$

$$\frac{\partial \mathbf{v_d}}{\partial t} + (\mathbf{v_d} \cdot \nabla)\mathbf{v_d} = \nabla \phi - \frac{\sigma}{n_d} \nabla P, \tag{3}$$

$$\frac{\partial P}{\partial t} + \mathbf{v_d} \cdot \nabla P + \gamma P \cdot \nabla \mathbf{v_d} = 0. \tag{4}$$

The system is closed by Poisson's equation

$$\nabla^2 \phi = n_e - n_i + n_d \tag{5}$$

where $\sigma = T_d/Z_d T_i$ is the normalized dust temperature, n_d is the dust density normalized by n_{d0} and $n_e(n_i)$ are the electron and ion number densities both normalized by $Z_d n_{d0}$, $\gamma = (2 + N)/N$ where N is the number of degrees of freedom, v_d is the dust fluid velocity normalized by the dust acoustic speed $C_d = (Z_d k_B T_i/m_d)^{1/2}$, P is the dust pressure normalized by $n_{d0} k_B T_d$, ϕ is the electrostatic wave potential normalized by $k_B T_i/e$ where e is the electronic charge, the space variable is normalized by $\lambda_d = (k_B T_i/4\pi Z_d n_{d0} e^2)^{1/2}$ and time is normalized by the inverse dust plasma frequency $\omega_{pd}^{-1} = (m_d/4\pi Z_d^2 n_{d0} e^2)^{1/2}$.

The ions are non-thermal and assumed to follow a distribution similar to the that used to describe non-thermal electrons by Cairns *et al.* [10] and the unnormalized ion density is given by

$$n_i = n_{i0}(1 + \beta\phi + \beta\phi^2)\,\mathrm{e}^{-\phi} \tag{6}$$

where

$$\beta = \frac{4\alpha}{1 + 4\alpha}. \tag{7}$$

The electrons are in thermal equilibrium. Their density is given by the Boltzmann expression

$$n_e = n_{e0}\,\mathrm{e}^{\delta\phi} \tag{8}$$

where $\delta = T_i/T_e$.

The equations in the one dimensional ($\gamma = 3$) case can be written as

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d v_d)}{\partial x} = 0, \tag{9}$$

$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} = \frac{\partial \phi}{\partial x} - \frac{\sigma}{n_d}\frac{\partial P}{\partial x}, \tag{10}$$

$$\frac{\partial P}{\partial t} + v_d \frac{\partial P}{\partial x} + 3P\frac{\partial v_d}{\partial x} = 0, \tag{11}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_d + \frac{n_{e0}}{Z_d n_{d0}}\mathrm{e}^{\delta\phi} - \frac{n_{i0}}{Z_d n_{d0}}(1 + \beta\phi + \beta\phi^2)\,\mathrm{e}^{-\phi}. \tag{12}$$

For solitary wave solutions, we transform to the stationary frame $\xi = x - Mt$ where M is the Mach number (the solitary wave velocity normalized by the dust acoustic speed C_d).

The above set of equations can now be written as

$$-M\frac{\partial n_d}{\partial \xi} + \frac{\partial (n_d v_d)}{\partial \xi} = 0, \tag{13}$$

$$-M\frac{\partial v_d}{\partial \xi} + v_d \frac{\partial v_d}{\partial \xi} = \frac{\partial \phi}{\partial \xi} - \frac{\sigma}{n_d}\frac{\partial P}{\partial \xi}, \tag{14}$$

$$-M\frac{\partial P}{\partial \xi} + v_d \frac{\partial P}{\partial \xi} + 3P\frac{\partial v_d}{\partial \xi} = 0, \tag{15}$$

$$\frac{\partial^2 \phi}{\partial \xi^2} = n_d - \frac{n_{i0}}{Z_d n_{d0}}(1 + \beta\phi + \beta\phi^2)\,\mathrm{e}^{-\phi} + \frac{n_{e0}}{Z_d n_{d0}}\mathrm{e}^{\delta\phi}. \tag{16}$$

By making use of the boundary conditions $\phi \to 0$, $v_d \to v_{d0}$ where v_{d0} is the equilibrium dust drift speed, $P \to 1$ and $n_d \to 1$ at $|\xi| \to \pm\infty$, equation (13) and (15) can be integrated to give

$$n_d = \frac{v_{d0} - M}{v_d - M} \tag{17}$$

and

$$P = n_d^3. \tag{18}$$

Combining (14) and (17), one gets

$$-2M\frac{\partial v_d}{\partial \xi} + 2v_d\frac{\partial v_d}{\partial \xi} + 2\sigma\frac{M}{M - v_{d0}}\frac{\partial P}{\partial \xi} - 2\sigma\frac{v_d}{M - v_{d0}}\frac{\partial P}{\partial \xi} = 2\frac{\partial \phi}{\partial \xi}. \tag{19}$$

Multiplying (15) by $\sigma/(M - v_{d0})$, we can write

$$-\frac{M}{M - v_{d0}}\sigma\frac{\partial P}{\partial \xi} + \frac{v_d}{M - v_{d0}}\sigma\frac{\partial P}{\partial \xi} + 3P\frac{\sigma}{M - v_{d0}}\frac{\partial v_d}{\partial \xi} = 0. \tag{20}$$

Subtracting (19) from (20), yields the following differential equation

$$2M\frac{\partial v_d}{\partial \xi} - 2v_d\frac{\partial v_d}{\partial \xi} - 3\sigma\frac{M}{M - v_{d0}}\frac{\partial P}{\partial \xi} + \left(\frac{3\sigma}{M - v_{d0}}\right)\frac{\partial (P v_d)}{\partial \xi} + 2\frac{\partial \phi}{\partial \xi} = 0. \tag{21}$$

Integrating (21), and using the same boundary conditions at $|\xi| \to \pm\infty$, we obtain

$$2M(v_d - v_{d0}) + ({v_{d0}}^2 - {v_d}^2) - 3P\sigma\frac{M}{M - v_{d0}} + 3\sigma P\frac{v_d}{M - v_{d0}} + 3\sigma\frac{M}{M - v_{d0}} - 3\sigma\frac{v_{d0}}{M - v_{d0}} + 2\phi = 0. \tag{22}$$

Substituting for v_d and P from (17) and (18), respectively, into this equation, we obtain a quadratic equation in n_d^2, viz.

$$3\sigma {n_d}^4 - ((M - v_{d0})^2 + 3\sigma + 2\phi){n_d}^2 + (M - v_{d0})^2 = 0. \tag{23}$$

The solution for n_d is given by:

$$n_d = \frac{\sigma_1}{\sqrt{2}\sigma_0} \times \sqrt{1 + \frac{2\phi}{(M - v_{d0})^2{\sigma_1}^2} - \sqrt{\left(1 + \frac{2\phi}{(M - v_{d0})^2{\sigma_1}^2}\right)^2 - 4\frac{\sigma_0^2}{\sigma_1^4}}} \tag{24}$$

where

$$\sigma_0 = \sqrt{\frac{3\sigma}{(M - v_{d0})^2}}; \quad \sigma_1 = \sqrt{1 + {\sigma_0}^2}. \tag{25}$$

It is important to note that when we set the normalized equilibrium dust drift velocity to zero ($v_{d0} = 0$) in (24) and (25), then our expressions for n_d, σ_0 and σ_1 correspond exactly with equations (22) and (23) in [14].

Substituting (24) in (16) gives

$$\frac{d^2\phi}{d\xi^2} = \frac{\sigma_1}{\sqrt{2}\sigma_0}\left[1 + \frac{2\phi}{(M - v_{d0})^2{\sigma_1}^2} - \sqrt{\left(1 + \frac{2\phi}{(M - v_{d0})^2{\sigma_1}^2}\right)^2 - 4\frac{\sigma_0^2}{\sigma_1^4}}\right]^{1/2} + \frac{n_{e0}}{Z_d n_{d0}} e^{\delta\phi} - \frac{n_{i0}}{Z_d n_{d0}}(1 + \beta\phi + \beta\phi^2)\, e^{-\phi}. \quad (26)$$

The nature of solutions of this equation is seen by introducing the Sagdeev potential [17]. Thus (26) can be rewritten in the form

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0 \quad (27)$$

where the Sagdeev potential $V(\phi)$ is given by

$$V(\phi) = -\frac{n_{i0}}{Z_d n_{d0}}(1 + 3\beta + 3\beta\phi + \beta\phi^2)\, e^{-\phi} - \frac{n_{e0}}{Z_d n_{d0}}\frac{1}{\delta}\, e^{\delta\phi} - (M - v_{d0})^2\sqrt{\sigma_0}\left(\exp\left[\frac{\theta}{2}\right] + \frac{1}{3}\exp\left[\frac{-3\theta}{2}\right]\right) + C_1 \quad (28)$$

and C_1 is chosen such that $V(\phi) = 0$ at $\phi = 0$.

As in Mendoza-Briceno *et al.* [14], to make it possible to consider the limit $\sigma \to 0$ in the Sagdeev potential, we express θ as

$$\theta = \ln\left[\frac{\sigma_1^2}{2\sigma_0^2}\left(1 + \frac{2\phi}{(M - v_{d0})^2\sigma_1^2}\right) + \sqrt{\frac{{\sigma_1}^4}{4{\sigma_0}^4}\left(1 + \frac{2\phi}{(M - v_{d0})^2{\sigma_1}^2}\right)^2 - 1}\right]. \quad (29)$$

Using (29), the Sagdeev potential can be expressed in the following form:

$$V(\phi) = -\frac{n_{i0}}{Z_d n_{d0}}(1 + 3\beta + 3\beta\phi + \beta\phi^2)\, e^{-\phi} - \frac{n_{e0}}{Z_d n_{d0}}\frac{1}{\delta}\, e^{\delta\phi} - \frac{(M - v_{d0})^2\sigma_1}{\sqrt{2}}\left[1 + \frac{2\phi}{(M - v_{d0})^2{\sigma_1}^2} + \sqrt{\left(1 + \frac{2\phi}{(M - v_{d0})^2{\sigma_1}^2}\right)^2 - 4\frac{\sigma_0^2}{\sigma_1^4}}\right]^{1/2} - \frac{2\sqrt{2}\sigma}{{\sigma_1}^3}\left[1 + \frac{2\phi}{(M - v_{d0})^2{\sigma_1}^2} + \sqrt{\left(1 + \frac{2\phi}{(M - v_{d0})^2{\sigma_1}^2}\right)^2 - 4\frac{\sigma_0^2}{\sigma_1^4}}\right]^{-3/2} + C_1. \quad (30)$$

If we set the electron density n_{e0} and dust drift speed v_{d0} to zero in (30), our expression for the Sagdeev potential agrees with equation (29) in ref. [14].

The solitary wave solutions of (27) exist if we have (*i*) $(d^2V/d\phi^2)_{\phi=0} < 0$ so that the fixed point at the origin is unstable, (*ii*) $V(\phi) < 0$ when $0 < \phi < \phi_{\max}$ for positive solitary waves and $\phi_{\min} < \phi < 0$ for negative solitary waves where $\phi_{\max(\min)}$ is the maximum (minimum) value of ϕ for which $V(\phi) = 0$ and (*iii*) $(d^3V/d\phi^3 > 0)_{\phi=0}$ (<0) for solitary waves with $\phi > 0$ (< 0).

The critical Mach number as defined in [14], is that which corresponds to the vanishing of the quadratic term in the Taylor expansion of the Sagdeev potential about the origin.

On expanding the Sagdeev potential $V(\phi)$ given by (30) around the origin, the critical Mach number at which the second derivative changes sign is given by

$$M_c = \sqrt{\frac{1 + 3\sigma\left[\frac{n_{i0}}{Z_d n_{d0}}(1 - \beta) + \frac{n_{e0}}{Z_d n_{d0}}\delta\right]}{\frac{n_{i0}}{Z_d n_{d0}}(1 - \beta) + \frac{n_{e0}}{Z_d n_{d0}}\delta}} + v_{d0}. \quad (31)$$

Then as discussed by Mendoza-Briceno [14], in addition to compressive solitons, the plasma would support rarefactive solitons (solitons of positive potential) if at M_c, we have $(d^3V(\phi)/d\phi^3)_{\phi=0} > 0$ i.e.

$$S_\alpha < 0 \quad (32)$$

where

$$S_\alpha = -\frac{n_{i0}}{Z_d n_{d0}} + \frac{n_{e0}}{Z_d n_{d0}}\delta^2 + \left\{3\left(\frac{n_{i0}}{Z_d n_{d0}}(\alpha - 1) - \frac{n_{e0}}{Z_d n_{d0}}(1 + 3\alpha)\delta\right)^2 \times\left[1 + 3\alpha + 4\sigma\left(\frac{n_{i0}}{Z_d n_{d0}} - \frac{n_{i0}}{Z_d n_{d0}}\alpha + \frac{n_{e0}}{Z_d n_{d0}}(1 + 3\alpha)\delta\right)\right]\right\}/(1 + 3\alpha)^3. \quad (33)$$

It is, however crucial to mention here that, when the conditions (31) and (32) are satisfied, we observe from numerical considerations of (30), that both compressive and rarefactive solitons are simultaneously supported in the non-thermal plasma as was reported by Mendoza-Briceno *et al.* [14] which was based on an earlier study by Cairns *et al.* [10] where it was first reported that a non-thermal plasma simultaneously supports solitons of positive and negative potential.

We note that if we set the normalized equilibrium electron number density to zero ($n_{e0} = 0$) and in the absence of dust streaming ($v_{d0} = 0$), then (31) becomes

$$M_c = \sqrt{\frac{1 + 3\sigma(1 - \beta)}{(1 - \beta)}} \quad (34)$$

which corresponds to the expression obtained by Mendoza-Briceno *et al.* [14] if we do a binomial expansion of $\sqrt{1 + 12\sigma(1 - \beta)}$ in the small σ limit.

The equations (31) and (32) give us the criteria for determining the cut-off conditions for which negative potential and positive potential solitary waves can simultaneously exist in a non-thermal plasma whereas similar criteria were used to determine the existence of a single soliton in a thermal dusty plasma in [18]. In the study involving a negatively charged dust fluid and Boltzmann ions, Mamun [12] found a condition similar to $S_\alpha > 0$ applying, yielding only compressive solitons.

In Figure 1 we consider a thermal plasma by setting the non-thermal coefficient $\alpha = 0$ in (30) and plot the Sagdeev potential against ϕ for different Mach numbers for two different values of the dust temperature. We observe that the thermal plasma supports only one type of soliton (with negative potential). This is keeping with the findings of Cairns *et al.* [10] where they found that a thermal plasma supported only a single type of soliton (with positive potential in this case) as opposed to when they

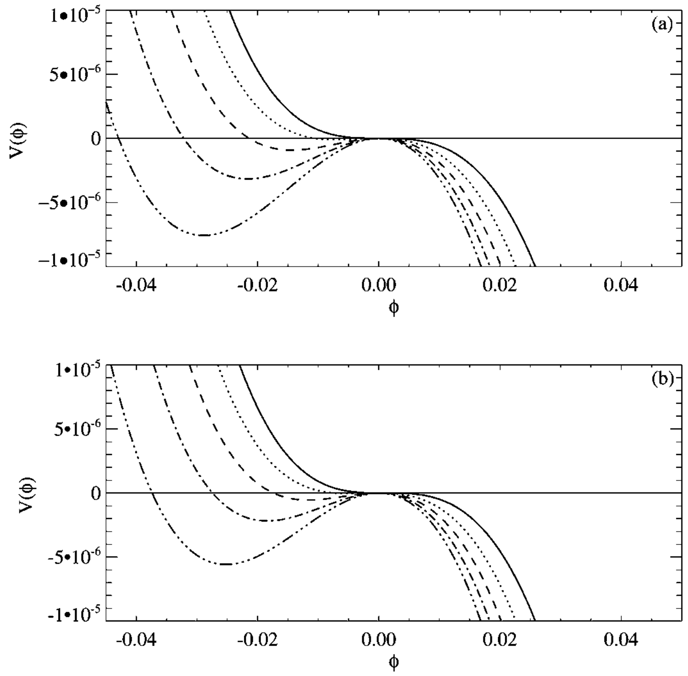

Fig. 1. The Sagdeev potential as a function of ϕ for (a) $\sigma = 0.02$ and (b) $\sigma = 0.04$. The curves correspond to (a) $M = 1.045$ (–), $M = 1.05$ $(\cdots)$, $M = 1.055$ (– – –), $M = 1.06$ (– · –) and $M = 1.065$ (– · · · –) and (b) $M = 1.075$ (–), $M = 1.08$ $(\cdots)$, $M = 1.085$ (– – –), $M = 1.09$ (– · –) and $M = 1.095$ (– · · · –). The other fixed parameters are $\alpha = 0$, $\delta = 1$, $n_{e0} = 0.1\ Z_d n_{d0}$ and $v_{d0} = 0.1\ C_d$.

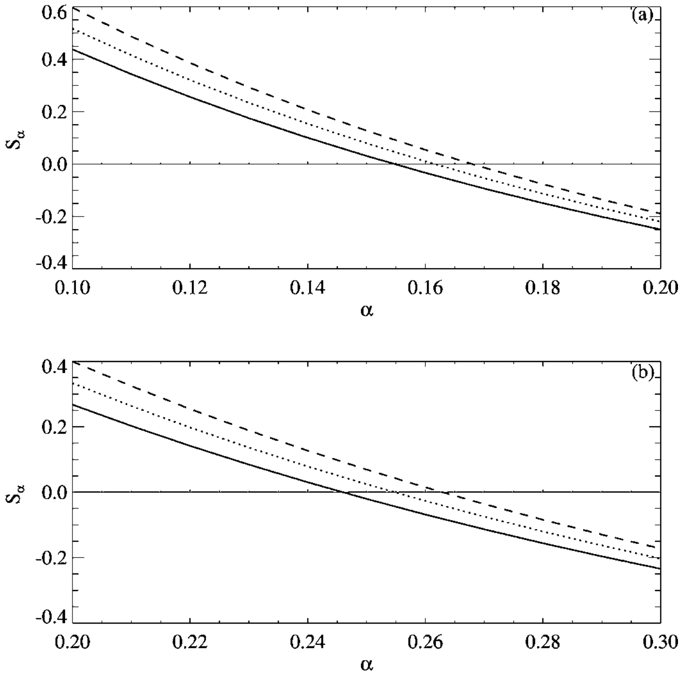

Fig. 2. The effect of dust temperature on the minimum value of α for which compressive and rarefactive solitons coexist for (a) $n_{e0}/Z_d n_{d0} = 0$ and (b) $n_{e0}/Z_d n_{d0} = 0.1$. The curves correspond to $\sigma = 0$ (–), $\sigma = 0.02$ $(\cdots)$ and $\sigma = 0.04$ (– – –). The other fixed parameter is $\delta = 1$.

considered a plasma with $\alpha \neq 0$. Also Mamun *et al.* [12] used a model comprising a negatively charged dust fluid and Boltzmann ions and found that the thermal plasma supported only negative potential solitary waves. The requirement that a plasma comprises one or more non-thermal particle constituents seems to be crucial in supporting the simultaneous existence of both positive and negative potential solitons.

We find that the critical Mach number of 1.045 calculated from (31) using the parameters of Figure 1(a) is in excellent agreement with the numerical result where we find that a compressive soliton starts to appear for $M = 1.05$ and also observe that $S_\alpha > 0$ is satisfied for this value of the Mach number. Thus rarefactive solitons do not exist in the thermal plasma.

In Figure 2 we consider a non-thermal plasma and observe that the minimum value of α required for the coexistence of rarefactive and compressive solitons is higher when there are electrons present in the plasma than when no electrons are present. We find that for a dust temperature $\sigma = 0$, the cut-off value of α required for the coexistence of two types of solitons increases from 0.155 when normalized $n_{e0} = 0$ (Figure 2(a)) to 0.246 when normalized $n_{e0} = 0.1$ (Figure 2(b)). The cut-off value $\alpha = 0.155$ that we observed for (32) to be satisfied, when we set $n_{e0} = 0$, is in agreement with that obtained by Mendoza-Briceno *et al.* in ref. [14] for the $\sigma = 0$ case. In the numerical work that follows, we set $\alpha = 0.3$ when we fix $n_{e0} = 0.1$.

In the presence of electrons ($n_{e0} = 0.1$), we examine the dependence of S_α on the dust temperature σ. The cutoff value of α required for the simultaneous existence of solitons for a fixed $n_{e0} = 0.1$ increases from 0.246 for a dust temperature $\sigma = 0$ to 0.255 when the dust temperature is $\sigma = 0.02$. In the presence of electrons, for the same dust temperature, a higher cut-off value of the non-thermal coefficient α is required for the non-thermal plasma to support the coexistence of two solitons when compared with the $n_{e0} = 0$ case in Mendoza-Briceno *et al.* [14]. We also observe that the required cutoff value of α for the coexistence of compressive and rarefactive solitons increases when the dust temperature increases for any fixed electron concentration, as found for $n_{e0} = 0$ in ref [14].

We study in Figure 3 the dependence of the critical Mach number on the non-thermal parameter α for different fixed parameters. We find that the dependence of M_c on α is linear as found by Mendoza-Briceno *et al.* [14]. We observe that the critical

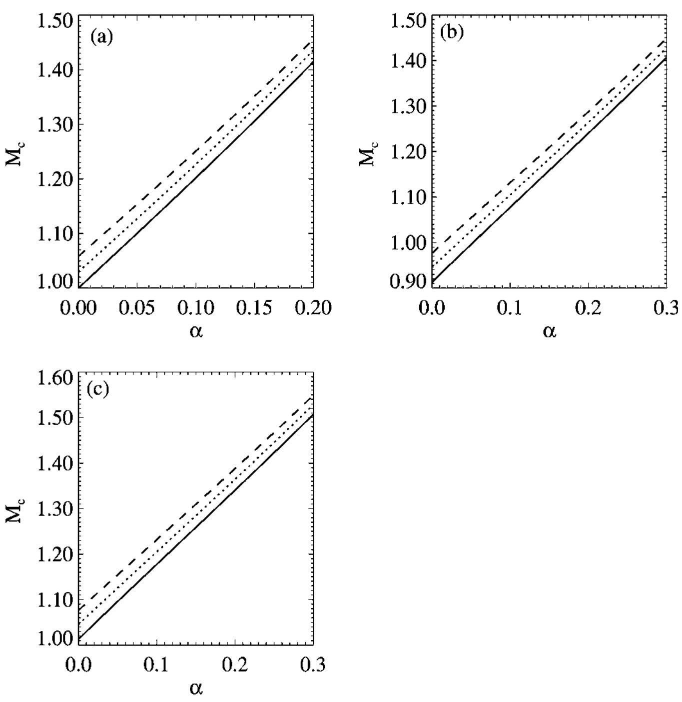

Fig. 3. The effect of dust temperature on the variation of the critical mach number as a function of α for (a) $n_{e0} = 0\ Z_d n_{d0}$, $v_{d0} = 0\ C_d$ (b) $n_{e0} = 0.1\ Z_d n_{d0}$, $v_{d0} = 0\ C_d$ and (c) $n_{e0} = 0.1\ Z_d n_{d0}$, $v_{d0} = 0.1\ C_d$. The curves correspond to $\sigma = 0$ (–), $\sigma = 0.02$ $(\cdots)$ and $\sigma = 0.04$ (– – –). The other fixed parameter is $\delta = 1$.

Mach number 1.414 calculated from (31) for $\sigma = 0$ and the other fixed parameters of Figure 3(a), is in agreement with that obtained by Mendoza-Briceno *et al.* [14] for the same dust temperature. We extend the scaling of the horizontal axis in Figures 3(b) and (c) to $\alpha = 0.3$ because we recall from our observations in Figure 2 that when we have normalized $n_{e0} = 0.1$ for a particular dust temperature, the cut-off value of α required for the coexistence of the solitons, is higher than when $n_{e0} = 0$ in the non-thermal plasma.

We observe that for a dust temperature $\sigma = 0$ and for any fixed value of α, say $\alpha = 0.3$, the critical Mach number for the parameters $n_{e0} = 0.1$ ($v_{d0} = 0$) in Figure 3(b) is 1.407 but $M_c = 1.507$ in Figure 3(c) when we have $n_{e0} = 0.1$ and when there is dust streaming ($v_{d0} = 0.1$). In the presence of electrons, the effect of streaming dust is to increase the critical Mach number that is required for the coexistence of rarefactive and compressive solitons.

For any one set of curves in Figure 3, for a fixed α, we observe that the effect of increasing the dust temperature σ is to increase the critical Mach number as was observed in by Mendoza-Briceno *et al.* [14]. For fixed $\alpha = 0.3$, we observe in Figure 3(c) that $M_c = 1.506$, 1.528, and 1.549 for $\sigma = 0, 0.02$ and 0.04, respectively.

In Figure 4 we plot the Sagdeev potential as a function of ϕ for two different dust temperatures $\sigma = 0.02$ (Figures 4(a) and (b)) and $\sigma = 0.04$ (Figure 4(c) and (d)) for a non zero electron concentration $n_{e0} = 0.1$. It is important to mention that when we set $n_{e0} = 0$ and $v_{d0} = 0$, our numerical results are found to be identical with that of Mendoza-Briceno *et al.* [14]. The value of the critical Mach number calculated from (31) for the parameters of Figure 4(a) and (b) for $\sigma = 0.02$ is 1.428 and we find that a rarefactive soliton starts to appear for $M = 1.43$. The value of the critical Mach number calculated from (31) for the parameters of Figure 4(c) and (d) for $\sigma = 0.04$ is 1.449 and we find that a rarefactive soliton starts to appear for $M = 1.45$. Compressive solitons are present for all values of the indicated Mach numbers for the $\sigma = 0.02$ and $\sigma = 0.04$ in Figure 4(a) and (c), respectively. Hence the theoretical prediction of the cut-off values of M_c required for the coexistence of rarefactive and compressive solitons calculated from (31) are in excellent agreement with numerical observations. We also observe that the condition (32) which is required to support the coexistence of both solitons in our non-thermal plasma, is satisfied.

We plot in Figure 5 the Sagdeev potential as a function of ϕ for two different dust temperatures $\sigma = 0.02$ (Figures 5(a) and (b)) and $\sigma = 0.04$ (Figure 5(c) and (d)) when we have dust streaming. The value of the critical Mach number calculated from (31) for the parameters of Figure 5(a) and (b) for $\sigma = 0.02$ is 1.528 and we find that a rarefactive soliton starts to appear for $M = 1.53$. The value of the critical Mach number calculated from (31) the parameters of Figure 5(c) and (d) for $\sigma = 0.04$ is 1.549 and we find that a rarefactive soliton starts to appear for $M = 1.55$. Compressive solitons are present for all values of the indicated Mach numbers for the $\sigma = 0.02$ and $\sigma = 0.04$ in Figure 5(a) and (c), respectively. A comparison of Figures 4 and 5 shows that M_c increases with v_{d0} for a fixed α.

In Figure 6 we study the dependence of the minimum value of $\alpha(\alpha_{\rm crit})$ for which we get coexistence of negative and positive potential solitary structures on the normalized electron concentration for three different dust temperatures. We observe that the dependence of $\alpha_{\rm crit}$ on normalized n_{e0} is nonlinear and increases as the electron concentration increases. For $n_{e0} = 0$ and dust temperature $\sigma = 0$, we recover the value of 0.155 for $\alpha_{\rm crit}$ which was obtained by Mendoza-Briceno [14]. We also observe that for any fixed value of the electron concentration, the value of $\alpha_{\rm crit}$ increases as the dust temperature increases.

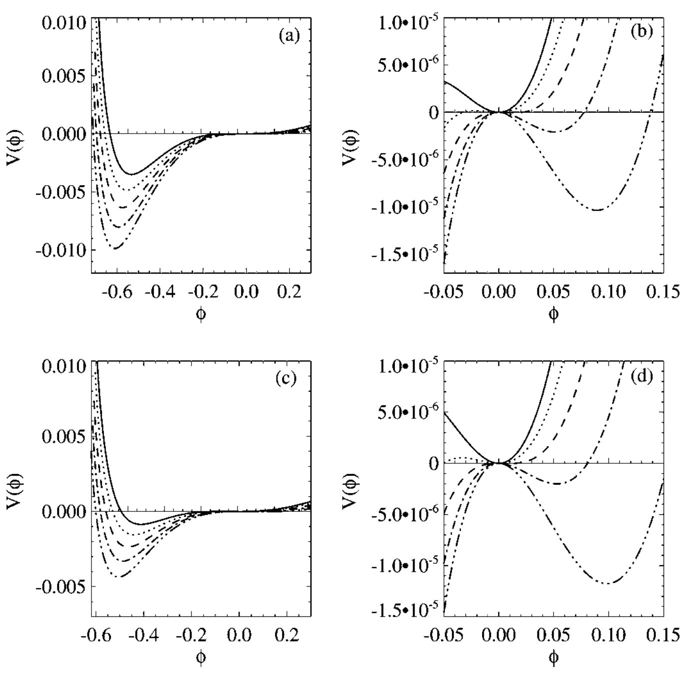

Fig. 4. The Sagdeev potential as a function of ϕ for $\sigma = 0.02$ ((a), (b)) and $\sigma = 0.04$ ((c), (d)). The curves ((a), (b)) are labelled by $M = 1.42$ (–), $M = 1.425$ (···), $M = 1.43$ (– – –), $M = 1.435$ (– · –) and $M = 1.44$ (– · · · –). The curves ((c), (d)) are labelled by $M = 1.44$ (–), $M = 1.445$ (···), $M = 1.45$ (– – –), $M = 1.455$ (– · –) and $M = 1.46$ (– · · · –). The fixed parameters are $\alpha = 0.3$, $n_{e0} = 0.1\ Z_d n_{d0}$, $\delta = 1$ and $v_{d0} = 0C_d$.

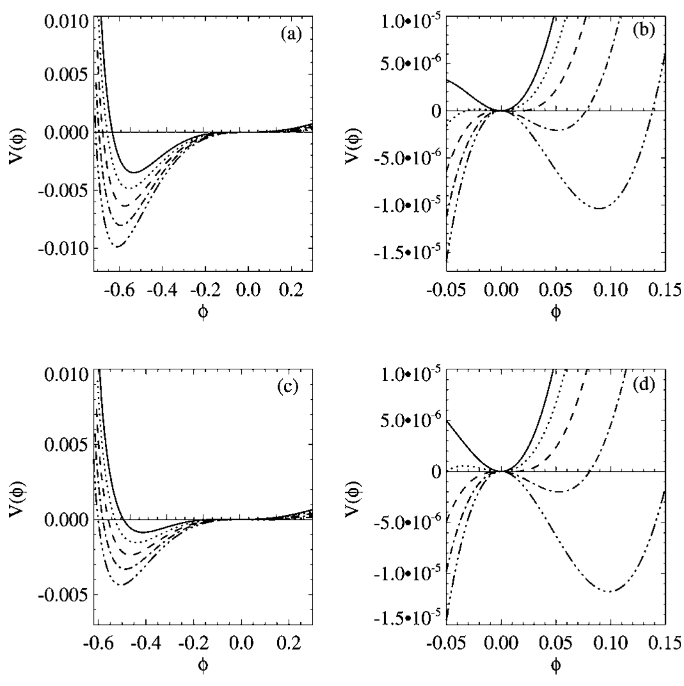

Fig. 5. The Sagdeev potential as a function of ϕ for $\sigma = 0.02$ ((a), (b)) and $\sigma = 0.04$ ((c), (d)). The curves ((a), (b)) are labelled by $M = 1.52$ (–), $M = 1.525$ (···), $M = 1.53$ (– – –), $M = 1.535$ (– · –) and $M = 1.54$ (– · · · –). The curves ((c), (d)) are labelled by $M = 1.54$ (–), $M = 1.545$ (···), $M = 1.55$ (– – –), $M = 1.555$ (– · –) and $M = 1.56$ (– · · · –). The fixed parameters are $\alpha = 0.3$, $n_{e0} = 0.1\ Z_d n_{d0}$, $\delta = 1$ and $v_{d0} = 0.1C_d$.

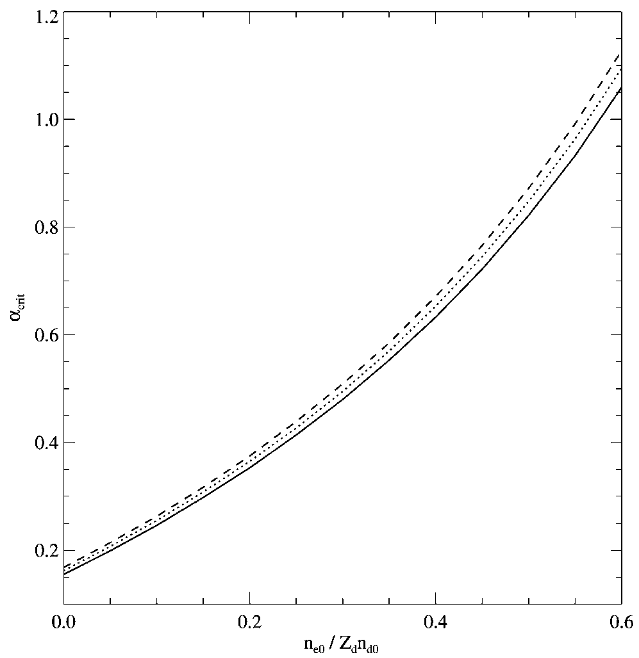

Fig. 6. The critical value of α as a function of $n_{e0}/Z_d n_{d0}$. The fixed parameter is $\delta = 1$. The curves correspond to $\sigma = 0$ (–), $\sigma = 0.02$ ($\cdots$) and $\sigma = 0.04$ (– – –).

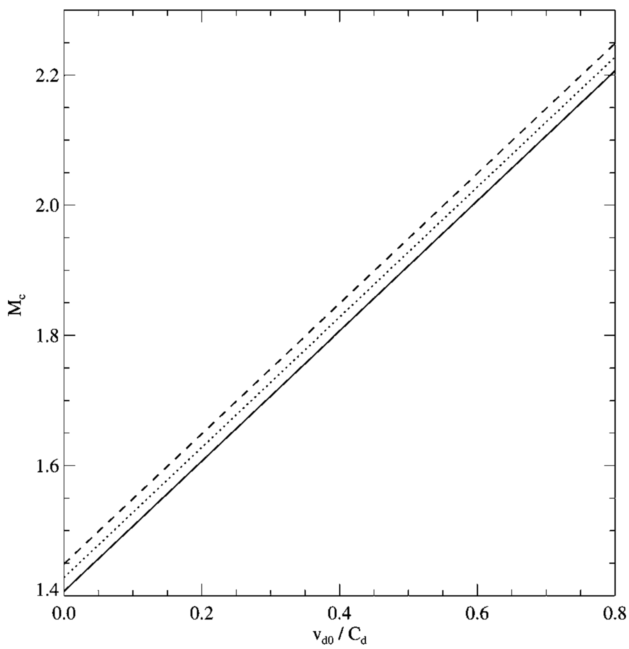

Fig. 7. The critical Mach number M_c as a function of v_{d0}/C_d. The fixed parameters are $n_{e0}/Z_d n_{d0} = 0.1$, $\alpha = 0.3$ and $\delta = 1$. The curves correspond to $\sigma = 0$ (–), $\sigma = 0.02$ ($\cdots$) and $\sigma = 0.04$ (– – –).

The dependence of M_c on the normalized dust drift velocity v_{d0} for three different dust temperatures is shown in Figure 7 for normalized n_{e0} =0.1. We observe that M_c increases linearly as v_{d0} increases. For any fixed value of v_{d0}, we observe that M_c increases as the dust temperature σ increases.

3. Conclusions

The effects of dust temperature and dust streaming on criteria which support the coexistence of compressive and rarefactive solitary waves, has been studied in an unmagnetized non-thermal dusty plasma. Our work is an extension of earlier investigations on dust ion plasmas (refs. [11, 12]) by the inclusion of a finite electron population. We have also incorporated the effect of drifting dust particles into our model since its a known fact that streaming can occur between charged particles in astrophysical and space environments.

We observe that when electrons are present in the non-thermal plasma, for a fixed dust temperature, a higher cut-off value of the non-thermal parameter α is required for both solitary waves to coexist, when compared with the $n_{e0} = 0$ case, which means that the plasma condition has to be further away from thermal equilibrium.

Our results show that when we have dust streaming, the critical Mach number required for the coexistence of positive and negative potential solitons is higher than when the dust is stationary for the same electron concentration. The minimum value of α required for coexistence of compressive and rarefactive solitons in the non-thermal plasma increases nonlinearly with the electron density n_{e0}. For a fixed electron concentration, the cut-off value of α required for the simultaneous existence of compressive and rarefactive solitons, increases as the dust temperature increases. On the other hand, the critical Mach number, is found to increase linearly with increasing v_{d0}. For any fixed value of v_{d0}, M_c increases as the dust temperature increases. Our results also show that the width of the solitons increase and the amplitudes decrease as the dust temperature increases. Our findings can be useful in understanding the behaviour of nonlinear potential structures in space and astrophysical environments where non-thermal ions are found to exist [5, 8, 9].

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