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Dust–acoustic nonlinear periodic waves in a dusty plasma with charge fluctuation

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Abstract. Using the reductive perturbation method, we present a theory for dust–acoustic nonlinear periodic waves, namely dust–acoustic cnoidal waves, in a plasma containing hot electrons and ions, and warm dust grains with varying charge. It is found that the frequency of the cnoidal wave is a function of its amplitude. It is also found that the dust grain charge fluctuation and other plasma parameters modify the properties of the dust–acoustic cnoidal waves. In the limiting case, these dust–acoustic cnoidal waves reduce to the well-known dust–acoustic solitons.

1. Introduction

There has been a great deal of interest in the study of dusty plasmas because of their importance in space environments, such as planetary rings, the interplanetary medium, cometary tails, the planetary magnetosphere and the lower part of the Earth's magnetosphere, the mesosphere, as well as laboratory plasmas (Goertz 1989; Northrop 1992; Mendis 2000; Verheest 2000; Shukla 2001; Shukla and Mamun 2002; Shukla and Eliasson 2009 and references therein). The importance of the study of dusty plasmas is rapidly growing in other areas of applications, such as plasma processing, thin-film deposition, microelectronics, production of complex materials, tokamaks and environmental research (Shukla 2001; Tsytovich et al. 2002; Shukla and Eliasson 2009). The dust grains are usually of micrometer or submicrometer size, and can be negatively charged by several mechanisms such as plasma currents, photoelectric emission, etc. In a recent colloquium, Shukla and Eliasson (2009) have presented an overview of some important features of dust–plasma interactions and discussed the underlying physics of several novel phenomena in dusty plasmas. A number of researchers have studied theoretically linear waves and nonlinear coherent localized structures in dusty plasmas (Rao et al. 1990; Bharuthram and Shukla 1992a, 1992b; Meuris and Verheest 1996). Several researchers observed a variety of wave modes in dusty plasmas (Barkan et al. 1995; Prabhakara and Tanna 1996; Merlino et al. 1998; Nakamura et al. 1999; Thomas and Merlino 2001). Barkan et al. (1995) and Thomas and Merlino (2001) reported experimental observations of very low frequency dust–acoustic waves. Nakamura et al. (1999) studied experimentally the dust-ion–acoustic waves in a dusty double-plasma device. A number of researchers have investigated experimentally the nonlinear coherent localized structures in dusty plasmas (Nakamura et al. 1999; Nakamura and

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Sarma 2001; Samsonov et al. 2002). Nakamura et al. (1999) observed the dust-ion–acoustic shocks in a dusty plasma. Nakamura and Sarma (2001) performed experimental observations of ion–acoustic solitary waves in dusty plasma in a dusty double-plasma device. Samsonov et al. (2002) reported the experimental observations of dissipative longitudinal solitons in a monolayer hexagonal dust lattice which was formed from monodisperse plastic microspheres and levitated in the sheath of an rf discharge.

In thermal plasmas, the dust grain charge is mainly affected by the plasma currents. Therefore, the dust grain charge is a variable and must be determined self-consistently by the electron and ion currents. Varma et al. (1993) studied the effect of dust charge fluctuations on electrostatic oscillations and found that the dust acoustic waves suffer a collisionless damping. Melandsø et al. (1993) showed that dust charge fluctuations lead to the damping of dust–acoustic waves in the limit when the dust charging frequency is smaller than the dust–acoustic frequency. Jana et al. (1993) showed that the effect of charge fluctuation leads to dissipative and instability mechanisms for ion waves in a dusty plasma. Rao and Shukla (1994) studied the nonlinear dust–acoustic waves in an unmagnetized plasma taking into consideration the dust grain charge perturbations. They showed that for unidirectional propagation, these waves are governed by the Korteweg–de Vries (KdV) equation with a source term. For the low-frequency case, they found the usual kind of KdV equation and discussed the localized soliton solution. Other authors (Popel et al. 1996; Melandsø 1996; Ma and Liu 1997; Singh and Rao 1998; Xie et al. 1999; Ghosh et al. 2000; Moolla et al. 2005) have also studied the effect of dust charge variation on the characteristics of the localized nonlinear structures. It has been shown by these authors that the dust charge fluctuations affect the properties of the localized nonlinear structures such as solitons and shock waves (Popel et al. 1996; Singh and Rao 1998; Xie et al. 1999; Ghosh et al. 2000). It must be noted that above-mentioned studies have been limited to bounded nonlinear structures.

There has been considerable interest in investigating the characteristics of nonlinear periodic waves in ordinary plasmas (Yadav et al. 1995; Tiwari et al. 2007 and references therein), because of their importance in the nonlinear transport processes in plasmas (Konno et al. 1979), ionospheric plasma (Gurevich and Stenflo 1988), explanation of single-mode drift wave spectra (Kauschke and Schlüter 1991), etc. Yadav et al. (1995) presented a comprehensive study of nonlinear periodic waves in an ordinary plasma with two electron species, which in the limiting case reduce to bounded nonlinear structures. The aim of this paper is to present a theory for nonlinear periodic waves in a dusty plasma with hot electrons and ions, and warm dust grains, considering the effect of dust charge variation, which in the limiting case reduce to dust–acoustic solitons. The plan of the paper is as follows. In Sec. 2 we give the basic equations of the system. In Sec. 3 we derive the KdV equation. In Sec. 4 we discuss the cnoidal wave solution of the KdV equation. Section 5 is devoted to the discussion of main results. Our main conclusions are summarized in Sec. 6.

2. Formulation of the problem

We consider a collisionless unmagnetized three-component dusty plasma consisting of hot electrons and ions, and massive negatively charged warm dust grains. The dynamics of one-dimensional very low frequency dust–acoustic waves in such a

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plasma is governed by the following basic equations:

$$\frac{\partial n_{\rm d}}{\partial t}+\frac{\partial}{\partial x}(n_{\rm d}u_{\rm d})=0, \tag{1}$$

$$\frac{\partial u_{\rm d}}{\partial t}+u_{\rm d}\frac{\partial u_{\rm d}}{\partial x}=\frac{q_{\rm d}}{m_{\rm d}}\frac{\partial \phi}{\partial x}-\frac{\gamma_{\rm d}T_{\rm d}}{m_{\rm d}n_{\rm d}}\frac{\partial n_{\rm d}}{\partial x}, \tag{2}$$

$$\frac{\partial q_{\rm d}}{\partial t}+u_{\rm d}\frac{\partial q_{\rm d}}{\partial x}=I_{\rm e}+I_{\rm i}, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2}=4\pi(en_{\rm e}-en_{\rm i}+q_{\rm d}n_{\rm d}), \tag{4}$$

where $n_{\rm d}, u_{\rm d}, q_{\rm d}, T_{\rm d}$ and $m_{\rm d}$ are, respectively, the number density, fluid velocity, magnitude of the negative charge, temperature in energy units and mass of the dust grains. $I_{\rm e}$ and $I_{\rm i}$ are, respectively, the electron and ion currents, given by (Varma et al. 1993; Jana et al. 1993; Rao and Shukla 1994)

$$I_{\rm e}=-\pi eR^2\left(\frac{8T_{\rm e}}{\pi m_{\rm e}}\right)^{1/2} n_{\rm e}\exp\left(\frac{e\Psi}{T_{\rm e}}\right), \tag{5}$$

$$I_{\rm i}=\pi eR^2\left(\frac{8T_{\rm i}}{\pi m_{\rm i}}\right)^{1/2} n_{\rm i}\left(1-\frac{e\Psi}{T_{\rm i}}\right), \tag{6}$$

where $m_{\rm e}$ $(m_{\rm i}), T_{\rm e}$ $(T_{\rm i})$ and $n_{\rm e}$ $(n_{\rm i})$ are the mass, temperature in energy units and number density of electrons (ions), respectively, and $\Psi=-q_{\rm d}/R$ is the dust grain surface potential relative to the plasma potential ϕ.

For very low frequency dust-acoustic waves, the electrons and ions are separately in thermal equilibrium and the electron number density $n_{\rm e}$ and ion number density $n_{\rm i}$ are described by the Boltzmann distribution

$$n_{\rm e}=n_{\rm e0}\exp\left(\frac{e\phi}{T_{\rm e}}\right), \tag{7}$$

$$n_{\rm i}=n_{\rm i0}\exp\left(-\frac{e\phi}{T_{\rm i}}\right), \tag{8}$$

where $n_{\rm e0}$ $(n_{\rm i0})$ is the equilibrium number density of electrons (ions).

Equations (1)–(8) constitute a complete set of basic equations governing the nonlinear dust–acoustic waves, considering the variable dust charge.

For many astrophysical and laboratory dusty plasmas, the dust charging time scale is much less than the dust hydrodynamical time scale; therefore, on the hydrodynamical time scale, the dust charge can reach local equilibrium, at which currents from ions and electrons to the dust grains are balanced (Xie et al. 1999). In this condition the current balance equation becomes

$$I_{\rm e}+I_{\rm i}\approx 0. \tag{9}$$

We normalize all the physical quantities as follows. Densities, time t, space coordinate x, dust fluid velocity $u_{\rm d}$, charge $q_{\rm d}=eZ_{\rm d}$ and potentials ϕ and Ψ are normalized by $n_{\rm d0}$, $\omega_{\rm pd}^{-1}=(m_{\rm d}/4\pi n_{\rm d0}Z_{\rm d0}^2e^2)^{1/2}$, $\lambda_{\rm Dd}=(T_{\rm eff}/4\pi Z_{\rm d0}n_{\rm d0}e^2)^{1/2}$, $C_{\rm d}=(Z_{\rm d0}T_{\rm eff}/m_{\rm d})^{1/2}$, unperturbed dust grain charge $q_{\rm d0}=eZ_{\rm d0}$ and $T_{\rm eff}/e$,

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respectively. The effective temperature T_{eff} is given by

$$T_{\mathrm{eff}} = \left(\frac{Z_{\mathrm{d0}} n_{\mathrm{d0}} T_{\mathrm{e}} T_{\mathrm{i}}}{n_{\mathrm{e0}} T_{\mathrm{i}} + n_{\mathrm{i0}} T_{\mathrm{e}}} \right). \tag{10}$$

Therefore, we obtain the following dimensionless basic equations from (1)–(4):

$$\frac{\partial n_{\mathrm{d}}}{\partial t} + \frac{\partial}{\partial x}(n_{\mathrm{d}} u_{\mathrm{d}}) = 0, \tag{11}$$

$$\frac{\partial u_{\mathrm{d}}}{\partial t} + u_{\mathrm{d}} \frac{\partial u_{\mathrm{d}}}{\partial x} = q_{\mathrm{d}} \frac{\partial \phi}{\partial x} - \frac{\gamma_{\mathrm{d}} \sigma_2}{n_{\mathrm{d}}} \frac{\partial n_{\mathrm{d}}}{\partial x}, \tag{12}$$

$$\alpha_{\mathrm{i}} \mu_1 (1 - \sigma \Psi) \exp(-\sigma \phi) - \exp(\sigma_1 \sigma (\Psi + \phi)) = 0, \tag{13}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{Z_{\mathrm{d0}}} (n_{\mathrm{e}} - n_{\mathrm{i}} + Z_{\mathrm{d}} n_{\mathrm{d}}), \tag{14}$$

where $\alpha_{\mathrm{i}} = (\sigma_1 m_{\mathrm{e}}/m_{\mathrm{i}})^{1/2}$, $\mu_1 = n_{\mathrm{i0}}/n_{\mathrm{e0}}$, $\sigma = T_{\mathrm{eff}}/T_{\mathrm{i}}$, $\sigma_1 = T_{\mathrm{i}}/T_{\mathrm{e}}$ and $\sigma_2 = T_{\mathrm{d}}/Z_{\mathrm{d0}} T_{\mathrm{eff}}$.

In obtaining (13), we used (5) and (6), with the condition (9).

The dimensionless density of electrons n_{e} and density of ions n_{i} in (14) are given, using (7) and (8), as

$$n_{\mathrm{e}} = n_{\mathrm{e0}} \exp(\sigma_1 \sigma \phi), \tag{15}$$

$$n_{\mathrm{i}} = n_{\mathrm{i0}} \exp(-\sigma \phi). \tag{16}$$

3. Derivation of the KdV equation

For the study of small but finite-amplitude dust–acoustic waves in the dusty plasma with variable dust charge, we derive the KdV equation from (11)–(14), using the reductive perturbation method. We introduce the stretched coordinates

$$\xi = \varepsilon^{1/2}(x - v_0 t), \quad \tau = \varepsilon^{3/2} t, \tag{17}$$

where ε is a small parameter and v_0 is the phase velocity of the wave. We expand the variable quantities about their equilibrium values in powers of ε:

$$\begin{aligned}
n_{\mathrm{d}} &= 1 + \varepsilon n_{\mathrm{d}}^{(1)} + \varepsilon^2 n_{\mathrm{d}}^{(2)} + \varepsilon^3 n_{\mathrm{d}}^{(3)} + \cdots, \\
u_{\mathrm{d}} &= \varepsilon u_{\mathrm{d}}^{(1)} + \varepsilon^2 u_{\mathrm{d}}^{(2)} + \varepsilon^3 u_{\mathrm{d}}^{(3)} + \cdots, \\
q_{\mathrm{d}} &= 1 + \varepsilon q_{\mathrm{d}}^{(1)} + \varepsilon^2 q_{\mathrm{d}}^{(2)} + \varepsilon^3 q_{\mathrm{d}}^{(3)} + \cdots, \\
\phi &= \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \varepsilon^3 \phi^{(3)} + \cdots.
\end{aligned} \tag{18}$$

Using (17) and (18) in (11)–(14), we find that the lowest order of (13) and (14) gives

$$\alpha_{\mathrm{i}} \mu_1 (1 - \sigma \Psi_0) - \exp(\sigma_1 \sigma \Psi_0) = 0, \tag{19}$$

$$n_{\mathrm{i0}} = n_{\mathrm{e0}} + Z_{\mathrm{d0}}. \tag{20}$$

Equations (19) and (20) represent current balance and quasi-neutrality conditions respectively in the equilibrium situation, in normalized form. The

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quasi-neutrality condition (20) in the dimensional form can be represented as $en_{i0} = en_{e0} + q_{d0}n_{d0}$ or $n_{i0} = n_{e0} + Z_{d0}n_{d0}$.

From the first-order equations, we get

$$n_{\rm d}^{(1)} = -(1+\gamma_1)\phi^{(1)}, \tag{21}$$

$$u_{\rm d}^{(1)} = -(1+\gamma_1)^{1/2}[1+\gamma_{\rm d}\sigma_2(1+\gamma_1)]^{1/2}\phi^{(1)} + C^{(1)}(\tau), \tag{22}$$

$$q_{\rm d}^{(1)} = \gamma_1\phi^{(1)}, \tag{23}$$

$$v_0 = (1+\gamma_1)^{-1/2}[1+\gamma_{\rm d}\sigma_2(1+\gamma_1)]^{1/2}, \tag{24}$$

where $C^{(1)}(\tau)$ is an integration constant that may depend on τ and

$$\gamma_1 = \frac{\gamma_{\rm b}}{\gamma_{\rm a}}\Psi_0. \tag{25}$$

$$\gamma_{\rm b} = -[\sigma_1\exp(\sigma_1\sigma\Psi_0) + \alpha_{\rm i}\mu_1(1-\sigma\Psi_0)] \tag{26}$$

$$\text{and}\quad \gamma_{\rm a} = \alpha_{\rm i}\mu_1(1+\sigma_1(1-\sigma\Psi_0)). \tag{27}$$

From (24), it is clear that the phase velocity of the dust–acoustic wave increases as we increase the temperature of the dust grains. It is consistent with Singh and Rao (1998). Thomas et al. (2007) have reported finite dust temperature effects on the characteristics of dust–acoustic waves. Their observations show that the phase velocity of dust–acoustic waves increases due to finite dust temperature effects.

Using the first-order solutions, we find that the second-order equations give

$$q_{\rm d}^{(2)} = \gamma_1\phi^{(2)} + \gamma_2\phi^{(1)^2}, \tag{28}$$

$$n_{\rm d}^{(2)} = \frac{\partial^2\phi^{(1)}}{\partial\xi^2} - (1+\gamma_1)\phi^{(2)} + \left\{\frac{(\mu_1-\sigma_1^2)\sigma^2}{2(\mu_1-1)} + \gamma_1(1+\gamma_1) - \gamma_2\right\}\phi^{(1)^2}, \tag{29}$$

$$\begin{aligned} u_{\rm d}^{(2)} = \frac{(1+\gamma_1)^{1/2}}{2[1+\gamma_{\rm d}\sigma_2(1+\gamma_1)]^{1/2}}\Bigg[&(1+\gamma_1)^{-1}\{1+2\gamma_{\rm d}\sigma_2(1+\gamma_1)\}n_{\rm d}^{(2)} \\ &-\left(\frac{1}{2}+\gamma_1+\gamma_{\rm d}\sigma_2(1+\gamma_1)^2\right)\phi^{(1)^2} - \phi^{(2)} + \frac{\partial C^{(1)}}{\partial\tau}\xi + C^{(2)}(\tau)\Bigg], \end{aligned} \tag{30}$$

where

$$\gamma_2 = \frac{\gamma_{\rm c}}{\gamma_{\rm a}\psi_0},$$

$$\gamma_{\rm c} = \alpha_{\rm i}\mu_1\sigma\psi_0\gamma_1 - \frac{\sigma_1^2\sigma}{2}(\psi_0\gamma_1+1)^2\exp(\sigma_1\sigma\psi_0) + \frac{\alpha_{\rm i}\mu_1(1-\sigma\psi_0)\sigma}{2}.$$

In (30), $C^{(2)}(\tau)$ is the second integration constant that may depend on τ. Imposing of a periodic boundary condition implies that

$$\frac{\partial C^{(1)}}{\partial\tau} = 0. \tag{31}$$

Therefore, $C^{(1)}$ is independent of ξ and τ.

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Using (29)–(31), we find that the second-order momentum equation gives the KdV equation

$$\frac{\partial \phi}{\partial \tau} + a\phi \frac{\partial \phi}{\partial \xi} + C^{(1)} \frac{\partial \phi}{\partial \xi} + b \frac{\partial^3 \phi}{\partial \xi^3} = 0, \tag{32}$$

where

$$b = \frac{(1+\gamma_1)^{-3/2}}{2[1+\gamma_{\rm d}\sigma_2(1+\gamma_1)]^{1/2}}, \tag{33}$$

$$a = b\left[\frac{(\mu_1 - \sigma_1^2)\sigma^2}{(\mu_1 - 1)} - 3(1+\gamma_1) - 2\gamma_2 - 2\gamma_{\rm d}\sigma_2(1+\gamma_1)^3\right] \tag{34}$$

and $\phi = \phi^{(1)}$.

4. Solution of KdV equation

For the steady-state solution of the KdV equation (32), we consider $\eta = \xi - u_1\tau$, where u_1 is a constant velocity.

Integrating (32) twice with respect to η, we obtain the so-called energy equation

$$\frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2 + V(\phi) = 0, \tag{35}$$

where the Sagdeev potential $V(\phi)$ is given by

$$V(\phi) = \frac{a}{6b}\phi^3 - \frac{u}{2b}\phi^2 + \rho_0\phi - \frac{1}{2}E_0^2, \tag{36}$$

where ρ_0 and E_0 are, respectively, the charge density and electric field when ϕ vanishes, and $u = u_1 - C^{(1)}$.

A cnoidal wave solution of (35) is given by (Yadav et al. 1995)

$$\phi = \alpha_2 + (\alpha_1 - \alpha_2)\mathrm{cn}^2\{D\eta, m\}, \tag{37}$$

where cn is the Jacobi elliptic function. The parameter m, called the modulus, and D can be expressed in terms of the three real zeros of the Sagdeev potential, α_1, α_2 and α_3, as

$$m^2 = \frac{\alpha_2 - \alpha_1}{\alpha_3 - \alpha_1}, \tag{38}$$

$$D = \sqrt{\frac{(\alpha_1 - \alpha_3)a}{12b}}. \tag{39}$$

For the dusty plasma considered here, we have $a < 0$ and α_1, α_2 and α_3 are such that $\alpha_1 < \alpha_2 \leqslant \alpha_3$.

The velocity u_1 is determined as

$$u_1 = C^{(1)} + \frac{a}{3}(\alpha_1 + \alpha_2 + \alpha_3). \tag{40}$$

The wavelength λ of the cnoidal wave is defined as

$$D\lambda = 2K(m), \tag{41}$$

where $K(m)$ is the complete elliptic integral of the first kind.

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Using the conservation condition of number density of dust grains

$$\int_0^\lambda (n_\mathrm{d} - 1)\, d\eta = 0, \tag{42}$$

we can find the three real zeros of the Sagdeev potential in terms of the modulus m. To evaluate (41), we assume that $n_\mathrm{d}^{(2)}$ and higher-order terms are very much less than $n_\mathrm{d}^{(1)}$.

Using (21), (37), (40) and (42), we obtain

$$u_1 = C^{(1)} - \frac{Aa}{3}\left\{\frac{1}{m^2}(2 - 3H(m)) - 1\right\}, \tag{43}$$

where $H(m) = E(m)/K(m)$ and $E(m)$ is the complete elliptic integral of the second kind. From (43), we get

$$A = -\frac{3(u_1 - C^{(1)})}{a\{(1/m^2)(2 - 3H(m)) - 1\}}. \tag{44}$$

The frequency, $f = V/\lambda$, of the cnoidal wave can be given by (38), (39), (41) and (42) as

$$f = \frac{V}{2K(m)m}\left[-\frac{aA}{12b}\right]^{1/2}. \tag{45}$$

The velocity V of the cnoidal wave in the laboratory frame is given by $V = v_0 + u_1$. Using (24) and (43), we have

$$V = (1+\gamma_1)^{-1/2}[1 + \gamma_\mathrm{d}\sigma_2(1+\gamma_1)]^{1/2} + C^{(1)} - \frac{Aa}{3}\left\{\frac{1}{m^2}(2 - 3H(m)) - 1\right\}. \tag{46}$$

5. Discussion

The coefficients a and b in the KdV equation (32) are functions of γ_1 and γ_2; therefore, from (44) and (45), it is clear that the amplitude and the frequency of the cnoidal wave are modified by the dust grain charge fluctuations. From (45), it is clear that the frequency of the cnoidal wave increases as the amplitude of the cnoidal wave increases. From (46), we find that for values of the modulus m below (above) a critical value m_c, the velocity V of the cnoidal wave decreases (increases) with amplitude. The critical value is found to be $m_\mathrm{c} \approx 0.949$.

In Fig. 1, we have plotted the Sagdeev potential $V(\phi)$ versus potential ϕ using (36), with a choice of $\mu_1 = 4, \sigma_1 = 0.1, \sigma_2 = 0$ and $u = 0.015$. In Fig. 2, we have plotted phase curves using (35). The dashed curve in Fig. 1 represents the general Sagdeev potential for the cnoidal wave solution, where it is seen that the pseudoparticle bounces back periodically in the potential well. This is also seen in the dashed phase curve (-------) in Fig. 2. The associated periodic nonlinear potential structure (dashed curve) is shown in Fig. 3. In the dashed phase curve of Fig. 2, we see that when $|E_0|$, $\rho_0 \neq 0$, the phase curve is repeated on the same path and one complete cycle corresponds to a wavelength in the physical space. In the mechanical analogy, whenever the pseudoparticle's velocity becomes zero (i.e. $d\phi/d\eta = 0$), the 'potential force' reflects it back since $-dV(\phi)/d\phi$ does not vanish, and therefore it oscillates between two 'points'. Consequently, the potential of the

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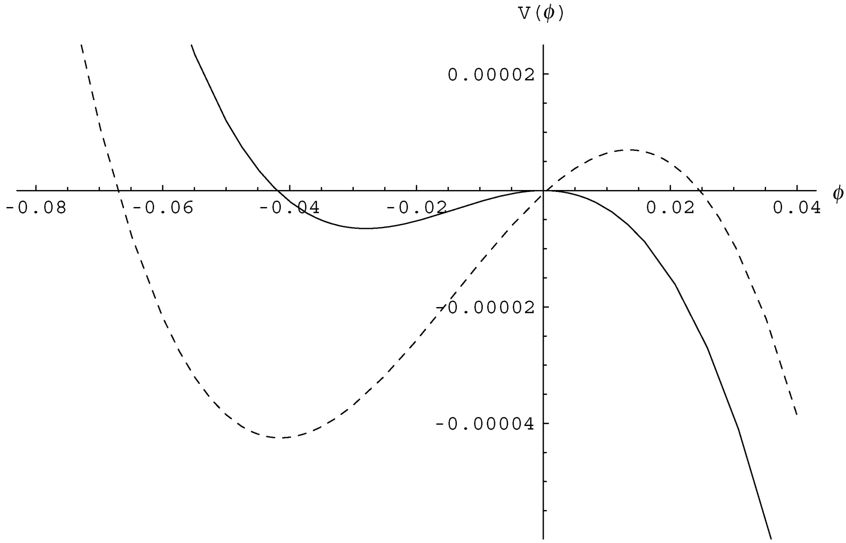

Figure 1. Plot of Sagdeev potential $V(\phi)$ versus potential ϕ using (36), with a choice $\mu_1 = 4, \sigma_1 = 0.1, m_{\rm i}/m_{\rm e} = 1846, \sigma_2 = 0$ and $u = 0.015$, with $E_0 = 0$, $\rho_0 = 0$ (———) and $|E_0| = 0.001$, $\rho_0 = 0.001$ (- - - - - - - - - - -).

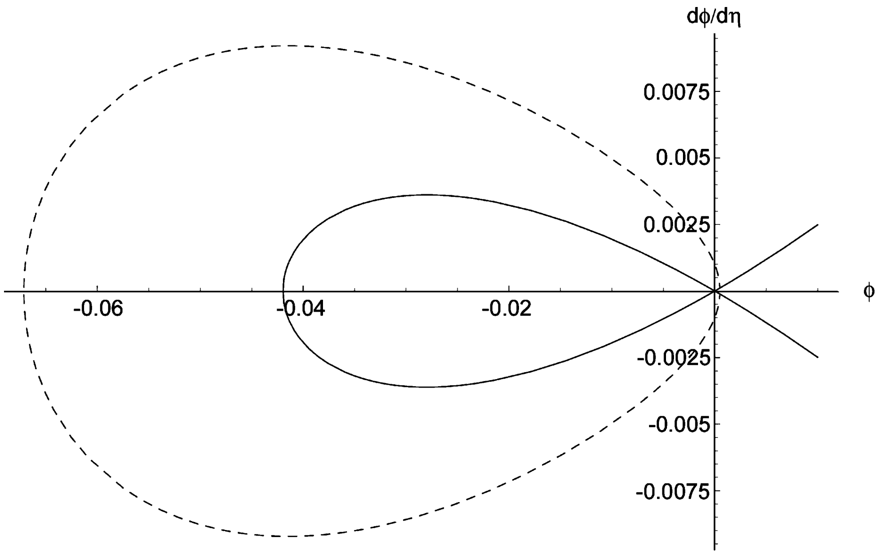

Figure 2. Phase curves using (35), with a choice $\mu_1 = 4, \sigma_1 = 0.1, m_{\rm i}/m_{\rm e} = 1846, \sigma_2 = 0$ and $u = 0.015$, with $E_0 = 0$, $\rho_0 = 0$ (———) and $|E_0| = 0.001$, $\rho_0 = 0.001$ (- - - - - - - - - - -).

cnoidal wave oscillates between the two lower values of the real zeros of the Sagdeev potential (see Figs 1 and 3).

In the limiting case, $m = 1$, $\alpha_2 = \alpha_3$ and the two right-hand solutions of the Sagdeev potential (dashed curve) merge at $\phi = 0$, to yield the normal Sagdeev potential (continuous curve) in Fig. 1 for a soliton structure, for which the corresponding

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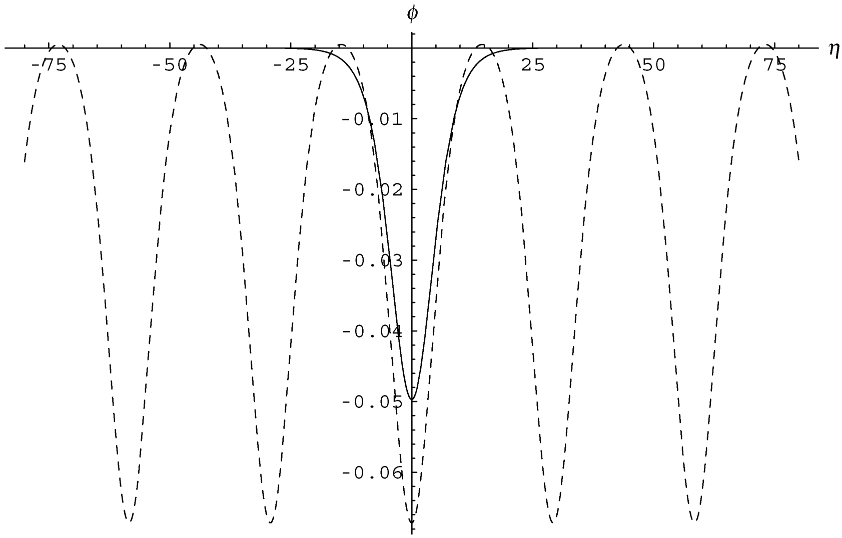

Figure 3. Variation of potential ϕ with space, with a choice $\mu_1 = 4$, $\sigma_1 = 0.1$, $m_i/m_e = 1846, \sigma_2 = 0$ and $u = 0.015$, with $E_0 = 0$, $\rho_0 = 0$ (———) using (37) and $|E_0| = 0.001$, $\rho_0 = 0.001$ (- - - - - - - - - - -) using (47).

curves are the continuous curves in Figs 2 and 3. In this case, $\alpha_1 = -A = 3u_1/a$ and cn = sech. This situation happens when $\rho_0 = E_0 = 0$. Therefore, in this case, the cnoidal wave solution (37) is reduced to the dust–acoustic soliton solution

$$\phi = \phi_m \ \mathrm{sech}^2(\eta/\delta), \tag{47}$$

where the amplitude of the soliton $\phi_m = 3u_1/a$ and width of the soliton $\delta = (4b/u_1)^{1/2}$. With $\sigma_2 = 0$, the dust–acoustic soliton solution (47) is reduced to that of Xie et al. (1999), for a dusty plasma with single ion species.

It must be emphasized that for the parameters considered here, the maximum amplitude of the solitary structures increases slightly with the inclusion of the dust charge variation; however, the width of the soliton decreases. Without the dust charge variation the maximum normalized amplitude and width come out to be −0.0393 and 11.55, respectively, whereas with charge fluctuations the normalized amplitude and width are −0.042 and 8.94, respectively. This is consistent with the findings of Singh and Rao (1998) for the nonlinear dust–acoustic waves for the case of the dust–acoustic wave frequency being smaller than the dust charging frequency.

When $E_0 = 0$ and $\rho_0 = 0$, the phase curve (continuous curve in Fig. 2) starts from the origin, circling clockwise around the negative ϕ axis, and again stops at the origin, entering from the upper side. In the ‘particle in a potential well analogy’, the pseudoparticle starts with zero speed at ‘position’ $\phi = 0$ and gains some speed along the negative ϕ axis; after attaining maximum speed, it again obtains zero speed. However, a ‘potential force’ reflects the pseudoparticle back toward the origin, and it gains speed in the opposite direction, i.e. along the positive ϕ axis, and again comes to rest at $\phi = 0$ (see continuous curves in Figs 1 and 2). In physical space, the value of the potential decreases from zero (at $\eta = -\infty$) to a minimum value at $\eta = 0$ and then starts increasing until it becomes zero (at $\eta = \infty$), and it represents

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a negative potential soliton (see continuous curve in Fig. 3), which is due to the compression of negative dust grains.

6. Conclusions

We have presented a theory for dust–acoustic nonlinear periodic waves, viz. dust–acoustic cnoidal waves, in a plasma consisting of hot electrons and ions, and warm dust grains with charge fluctuation. It is found that the frequency of the cnoidal wave depends on its amplitude, as the amplitude of the cnoidal wave increases its frequency also increases. It is found that the frequency and the amplitude of the cnoidal wave depend on the dust grain charge variations. In the limiting case, these dust–acoustic cnoidal waves reduce to dust–acoustic solitons. The maximum amplitude of the solitary structures increases slightly with the inclusion of the dust charge variation; however, the width of the soliton decreases. We have explained the dust–acoustic cnoidal waves and dust–acoustic solitons in terms of physical parameters depicting the Sagdeev potential curves and phase-space curves. It is plausible that the periodic signals may appear in astrophysical dusty plasmas. We infer that the present theory may be useful to explain the periodic signals as well as localized solitary structures in space environments, where dusty plasmas exist.

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References

Barkan, A., Merlino, R. L. and D'Angelo, N. 1995 Laboratory observation of the dust-acoustic wave mode. *Phys. Plasmas* **2**, 3563.

Bharuthram, R. and Shukla, P. K. 1992a Large amplitude double layers in dusty plasmas. *Planet. Space Sci.* **40**, 465.

Bharuthram, R. and Shukla, P. K. 1992b Vortices in non-uniform dusty plasmas. *Planet. Space Sci.* **40**, 647.

Ghosh, S., Sarkar, S., Khan, M. and Gupta, M. R. 2000 Dust ion acoustic shock waves in a collisionless dusty plasma. *Phys. Lett.* A **275**, 109.

Goertz, C. K. 1989 Dusty plasmas in the solar system. *Rev. Geophys.* **27**, 271.

Gurevich, A. V. and Stenflo, L. 1988 Nonlinear defocusing of radio wave beams in the ionosphere. *Phys. Scr.* **38**, 855.

Jana, M. R., Sen, A. and Kaw, P. K. 1993 Collective effects due to charge-fluctuation dynamics in a dusty plasma. *Phys. Rev.* E **48**, 3930.

Kauschke, U. and Schlüter, H. 1991 On nonlinear periodic drift waves. *Plasma Phys. Control. Fusion* **33**, 1309.

Konno, K., Mitsuhashi, T. and Ichikawa, Y. H. 1979 Propogation of ion acoustic cnoidal wave. *J. Phys. Soc. Japan* **46**, 1907.

Ma, J. X. and Liu, J. 1997 Dust-acoustic soliton in a dusty plasma. *Phys. Plasmas* **4**, 253.

Melandsø, F. 1996 Lattice waves in dust plasma crystals. *Phys. Plasmas* **3**, 3890.

Melandsø, F., Aslaksen, T. K. and Havens, O. 1993 A new damping effect for the dust-acoustic wave. *Planet. Space Sci.* **41**, 321.

Mendis, D. A. 2000 Of comets, rings and other things: A random walk in cosmic physics. *Planet. Space Sci.* **48**, 251.

https://doi.org/10.1017/S002237780900806X Published online by Cambridge University Press

Merlino, R., Barkan, A., Thompson, C. and D'Angelo, N. 1998 Laboratory studies of waves and instabilities in dusty plasmas. *Phys. Plasmas* **5**, 1607.

Meuris, P. and Verheest, F. 1996 Korteweg-de Vries equation for magnetosonic modes in dusty plasmas. *Phys. Lett.* A **219**, 299.

Moola, S., Bhauthram, R. and Baboolal, S. 2005 Reduction of dust acoustic soliton amplitude due to dust charge variation. *Phys. Plasmas* **12**, 042310.

Nakamura, Y., Bailung, H. and Shukla, P. K. 1999 Observation of ion-acoustic shocks in a dusty plasma. *Phys. Rev. Lett.* **83**, 1602.

Nakamura, Y. and Sarma, A. 2001 Observation of ion-acoustic solitary waves in a dusty plasma. *Phys. Plasmas* **8**, 3921.

Northrop, T. G. 1992 Dusty plasmas. *Phys. Scr.* **45**, 475.

Popel, S. I., Yu, M. Y. and Tsytovich, V. N. 1996 Shock waves in plasmas containing variable–charge impurities. *Phys. Plasmas* **3**, 4313.

Prabhakara, H. R. and Tanna, V. L. 1996 Trapping of dust and dust acoustic waves in laboratory plasmas. *Phys. Plasmas* **3**, 3176.

Rao, N. N. and Shukla, P. K. 1994 Nonlinear dust-acoustic waves with dust charge fluctuations. *Planet. Space Sci.* **42**, 221.

Rao, N. N., Shukla, P. K. and Yu, M. Y. 1990 Dust-acoustic waves in dusty plasmas. *Planet. Space Sci.* **38**, 543.

Samsonov, D., Ivlev, A. V., Quinn, R. A., Morfill, G. and Zhdanov, S. 2002 Dissipative longitudinal solitons in a two-dimensional strongly coupled complex (dusty) plasma. *Phys. Rev. Lett.* **88**, 095004.

Shukla, P. K. 2001 A survey of dusty plasma physics. *Phys. Plasmas* **8**, 1791.

Shukla, P. K. and Eliasson, B. 2009 Fundamentals of dust-plasma interactions. *Rev. Mod. Phys.* **81**, 25.

Shukla, P. K. and Mamun, A. A. 2002 *Introduction to Dusty Plasma Physics*. Institute of Physics Publishing, Bristol, pp. 1–35.

Singh, S. V. and Rao, N. N. 1998 Adiabatic dust-acoustic waves with dust-charge fluctuations. *J. Plasma Phys.* **60**, 541.

Thomas, Jr., E., Fisher, R. and Merlino, R. L. 2007 Observations of dust acoustic waves driven at high frequencies: Finite dust temperature effects and wave interference. *Phys. Plasmas* **14**, 123701.

Thomas, Jr., E. and Merlino, R. L. 2001 Dust particle motion in the vicinity of dust acoustic waves. *IEEE Trans. Plasma Sci.* **29**, 152.

Tiwari, R. S., Jain, S. L. and Chawla, J. K. 2007 Ion acoustic cnoidal waves and associated nonlinear ion flux in a warm ion plasma. *Phys. Plasmas* **14**, 022106.

Tsytovich, V. N., Morfill, G. E. and Thomas, H. 2002 Complex Plasmas: I. Complex plasmas as unusual state of matter. *Plasma Phys. Rep.* **28**, 623.

Varma, R. K., Shukla, P. K. and Krishan, V. 1993 Electrostatic oscillations in the presence of grain-charge perturbations in dusty plasmas. *Phys. Rev.* E **47**, 3612.

Verheest, F. 2000 *Waves in Dusty Space Plasmas*. Kluwer Academic, Dordrecht, pp. 41–66.

Xie, B., He, K. and Huang, Z. 1999 Dust-acoustic solitary waves and double layers in dusty plasma with variable dust charge and two-temperature ions. *Phys. Plasmas* **6**, 3808.

Yadav, L. L., Tiwari, R. S., Maheshwari, K. P. and Sharma, S. R. 1995 Ion-acoustic nonlinear periodic waves in a two-electron-temperature plasma. *Phys. Rev.* E **52**, 3045.

https://doi.org/10.1017/S002237780900806X Published online by Cambridge University Press