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Effect of hot ion temperature on obliquely propagating ion-acoustic solitons and double layers in an auroral plasma

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ABSTRACT

Properties of obliquely propagating ion-acoustic solitons and double layers in a magnetized auroral plasma composed of hot adiabatic ions and two types of, cool and hot Maxwellian electrons are studied using Sagdeev pseudo-potential technique and assuming the quasi-neutrality condition. The new and surprising result which emerges from the model is that in contrast to the case of cold ions where ion-acoustic solitons and double layers are found for subsonic Mach numbers only, the hot ions case allows these nonlinear structures to exist for both subsonic and supersonic Mach number regimes. The double layers exist at lower angle of propagation as hot ion temperature is increased. The soliton electric field amplitudes are increased but their width and pulse duration are decreased with the increase in hot ion temperature. For the auroral zone parameters, the maximum electric field amplitude, width, pulse duration and speed for the solitons come out to be in the range $\sim$ (0.3–15) mV/m, $\sim$ (195–455) m, (7–20) ms and (22–26) km/s, respectively. The results seem to be in agreement with the Viking satellite observations in the auroral zone.

1. Introduction

The fact that space is occupied with nonlinear wave phenomena has attracted many researchers. Soliton and double layer formation and propagation are among the most interesting and challenging problems in space plasmas. Washimi and Taniuti [1] investigated the properties of the Kortweg-deVries (KdV) equation of small amplitude nonlinear ion-acoustic solitary waves in a cold collisionless plasma. Using the fluid equations, Sagdeev [2] described the properties of solitons in a plasma consisting of hot isothermal electrons and cold ions. More extensive investigations on laboratory experiment [3,4] and theoretical analysis [5–9] have been done on the existence and behaviours of ion-acoustic solitary waves in various plasmas. Later, the observations of solitary waves propagating along the auroral magnetic field lines were reported by the several spacecraft missions: S3-3 [10], Viking [11], FREJA [12], POLAR [13,14] and FAST [15–17] satellites.

The existence of ion-acoustic solitary waves and double layers in a plasma consisting of two electron temperatures have been investigated by many authors in the past [18–23]. Small amplitude ion-acoustic double layers and solitons in auroral magnetized plasma consisting of hot and cold electrons and two ions (oxygen-hydrogen) have been studied by Reddy and Lakhina [24]. Their analysis predicted either negative potential double layers, or negative potential solitons or positive potential solitons in distinct parametric space. Reddy et al. [25] extended the analysis to include any number of ion beams and found that the fast and slow hydrogen (as well as oxygen) beam-acoustic modes can be generated which can be either

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rarefactive double layers or rarefactive or compressive solitons. Sayal et al. [26] studied the propagation characteristics of ion-acoustic rarefactive soliton in a two electron temperature unmagnetized plasma with two cold ions, using kinetic and fluid models for both the ion species. Motivated by the measurements performed by FREJA [12] and Swedish Viking satellites at low altitudes region of auroral plasma, Cairns et al. [27] predicted the coexistence of ion-acoustic solitons with both density peaks and density depressions in plasma consisting of ions and electrons. Berthomier et al. [28] described the characteristics of ion acoustic solitary waves and weak double layers in the presence of two electron species in an unmagnetized plasma with finite ion temperature. Using a model consistent with satellite observations, Kourakis and Shukla [29] presented the theoretical and numerical analysis for ion-acoustic solitary waves in an unmagnetized, collisionless, three-component plasma, composed of positive cold ion fluid moving in a background of two thermalized electron population. Ghosh and Lakhina [30] exploited the measurements by POLAR satellite to explain the width-amplitude variation of rarefactive ion-acoustic solitary waves in the auroral plasma. Employing the Sagdeev pseudo-potential technique, they investigated analytically solitary structures in both unmagnetized and magnetized multi-ion plasmas with two electrons temperature. Verheest et al. [31] discussed the necessary conditions for large amplitude ion acoustic solitons in unmagnetized multi-ion plasma with two electron temperatures. Recently, Lakhina et al. [32,33] highlighted the possibilities of the existence of three types of solitary waves, depending on the Mach number regime e.g., slow ion-acoustic, ion-acoustic and electron-acoustic solitons. These were predicted in their proposed model for arbitrary amplitude nonlinear ion- and electron-acoustic solitary waves in an unmagnetized multi-component plasma consisting of cold background electrons and ions, hot electron and ion beams. In a recent study, Lakhina et al. [34] showed the possibilities of ion-acoustic, slow and fast electron-acoustic soliton/double layer, depending upon the plasma parameters. The results were related to the observations of the electrostatic solitary waves (ESWs) observed in the Earth's magnetosphere by the Cluster satellite. More recently, Rufai et al. [35] extended the work of Berthomier et al. [28] and Baluku et al. [36] by assuming magnetized ion species. They investigated the finite amplitude ion-acoustic solitary wave and double layer structures in a magnetized plasma consisting of cold ions and two-temperature electrons and applied their results to Viking satellite observations in the auroral region.

In this paper, we investigate electrostatic solitary waves and double layer structures in a magnetized plasma consisting of an adiabatic ions fluid and two Maxwellian electron populations. Our present investigation extends the recent work of Rufai et al. [35] by including the effect of a finite ion temperature through the adiabatic condition. The inclusion of finite ion temperature pushes the limits of both minimum and maximum Mach numbers for solitons to higher sides. Section 2 present the basic set of equations governing the dynamics of plasma system. In Section 3, we obtain the localized solution of nonlinear finite amplitude waves, using the Sagdeev potential technique for solitons and double layers. Detailed numerical results are presented in Section 4 and discussion and conclusions are drawn in Section 5.

2. Theoretical model

We consider the collisionless, homogeneous plasma with adiabatic ions and two temperature electrons, in presence of an external static magnetic field ($B_o = B_o\hat{z}$), along the z-direction and waves are propagating in the (x, z)-plane. The two group of electrons, cool electrons (N_c, T_c) and hot electrons (N_h, T_h) are assumed to follow Boltzmann distribution, i.e.,

$$N_c = N_{c0} \exp\left(\frac{e\phi}{T_c}\right) \tag{1}$$

$$N_h = N_{h0} \exp\left(\frac{e\phi}{T_h}\right) \tag{2}$$

while the adiabatically heated ions are governed by the fluid equations. The phase velocity of the ion-acoustic wave is assumed much less than the thermal velocities of the cool and hot electrons, i.e., $\frac{\omega}{k} \leqslant v_{tc},\ \ v_{th}$, where $v_{tc,h} = (T_{c,h}/m_e)^{1/2}$ are thermal velocities of the cold and hot electrons, m_e is the electron mass. Then, the adiabatic ions are described by the fluid equations given below,

Continuity equation:

$$\frac{\partial N_i}{\partial t} + \nabla.(N_i\mathbf{V_i}) = 0. \tag{3}$$

Momentum equation:

$$\frac{\partial \mathbf{V_i}}{\partial t} + \mathbf{V_i}.\nabla\mathbf{V_i} = -\frac{e\nabla\phi}{m_i} + e\frac{\mathbf{V_i}\times\mathbf{B_o}}{m_i c} - \frac{1}{N_i m_i}\nabla P_i. \tag{4}$$

Pressure equation:

$$\frac{\partial P_i}{\partial t} + \mathbf{V_i}.\nabla P_i + \gamma P_i.\nabla\mathbf{V_i} = 0 \tag{5}$$

where N_i V_i and m_i are the number density, fluid velocity and mass of the ions, e is the magnitude of the electron charge, c is the speed of the light in vacuum and the ion pressure P_i is given by the balance pressure Eq. (5). further, ion pressure can be written as yields

$$P_i = P_{io}\left(\frac{N_i}{N_{io}}\right)^{\gamma}, \tag{6}$$

where $\gamma = \frac{(N+2)}{N}$, is adiabatic index and N is the number of degrees of freedom. For magnetized adiabatic ions $N = 3$, hence γ=5/3 and the ion pressure at equilibrium is $P_{io} = N_{io}T_i$.

For the three species plasma, the quasi-neutrality condition at equilibrium is given by $N_{io} = N_{co} + N_{ho} = N_o$. We use the following normalizations: density is normalized by total electron (ion) density, N_o , velocity is normalized by the effective ion acoustic speed $c_s = (T_{eff}/m_i)^{1/2}$, electrostatic potential, ϕ is normalized by T_{eff}/e, distance by effective ion Larmor radius, $\rho_i = c_s/\Omega$, time t normalized by inverse of ion gyro-frequency Ω^{-1}, ($\Omega = eB_o/m_ic$). We also define the cool to hot electron temperature ratio $\tau = T_c/T_h$, cool electron density ratio $f = N_{co}/N_o$, where $N_{jo}(j = c, h, i)$, $T_{eff} = T_c/(f + (1-f)\tau)$ is the effective electron temperature, $\alpha_c = T_{eff}/T_c$, $\alpha_h = T_{eff}/T_h$, $\sigma = T_i/T_{eff}$, where T_i is the ion temperature.

We then present the set of Eq. (1)–(5) in normalized form:

$$n_c = f\exp(\alpha_c\psi) \tag{7}$$

$$n_h = (1-f)\exp(\alpha_h\psi) \tag{8}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i v_x)}{\partial x} + \frac{\partial(n_i v_z)}{\partial z} = 0 \tag{9}$$

$$\frac{\partial v_x}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_x = -\frac{\partial\psi}{\partial x} + v_y - \frac{\sigma}{n_i}\frac{\partial}{\partial x}(n_i)^{5/3} \tag{10}$$

$$\frac{\partial v_y}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_y = -\frac{\partial\psi}{\partial y} - v_x \tag{11}$$

$$\frac{\partial v_z}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_z = -\frac{\partial\psi}{\partial z} - \frac{\sigma}{n_i}\frac{\partial}{\partial z}(n_i)^{5/3} \tag{12}$$

then, the set of the equations are closed with the quasi-neutrality condition

$$n_i = n_c + n_h = f\exp(\alpha_c\psi) + (1-f)\exp(\alpha_h\psi) \tag{13}$$

where n_j is normalized density of j^{th} species and $\psi = e\phi/T_{eff}$ is the normalized electrostatic potential.

We would like to emphasize here that the solitary waves in multi-component magnetized plasmas can be studied by adopting two approaches to arrive at an analytical solution:

1. Using the Poisson equation and carrying out the analysis with small amplitude theory and arrive at soliton solution through Kortweg-de-Vries (KdV) equation.
2. In low-frequency case such as ion-acoustic waves studied here, one can use the quasi-neutrality condition to arrive at the analytical solitary wave solution. However, the results are valid only for wavelengths longer than the Debye length.

In this paper we have adopted the second approach which has been used by many authors [35,37–41] while studying obliquely propagating nonlinear electrostatic waves in magnetized plasmas.

3. Localized stationary solution

For arbitrary amplitude treatment we look for soliton structures in a reference frame moving with the wave, i.e., with Mach number. Assuming that all the dependent variables depend on a single independent variable ξ such that,

$$\xi = (\alpha x + \beta z - Mt)/M \tag{14}$$

where $M = V/c_s$, and $V = \omega/k$ is the wave phase speed and $\alpha = \sin\theta, \beta = \cos\theta; \theta$ is the angle of the wave propagation relative to $\vec{B}_0$.

Transforming the ion continuity and momentum equations in (9)–(12) in terms of the coordinate ξ and integrating with appropriate boundary conditions for solitary wave structure (namely, $n_i \to 1, \psi \to 0$, and $d\psi/d\xi \to 0$ at $\xi \to \pm\infty$) , we obtain a single dimensionless nonlinear differential equation in terms of the ion density n_i and electrostatic potential ψ as,

$$\frac{d}{d\xi}\left(\frac{d\chi(\psi)}{d\xi}\right) = M^2(n_i - 1) + \beta^2\sigma n_i(1 - n_i^{5/3}) - \beta^2 n_i\left(\frac{f}{\alpha_c}(e^{\alpha_c\psi} - 1) + \frac{1-f}{\alpha_h}(e^{\alpha_h\psi} - 1)\right) \tag{15}$$

where

$$\chi(\psi) = \left(\psi + \frac{M^2}{2n_i^2} + \frac{5}{2}\sigma n_i^{2/3}\right) \tag{16}$$

with n_i given by Eq. (13).

Now multiplying both sides of Eq. (15) by $d\chi/d\xi$ and integrating once with appropriate boundary conditions, we obtain

$$\frac{1}{2}\left(\frac{d\chi(\psi)}{d\xi}\right)^2 + V(\psi, M) = 0 \tag{17}$$

From Eqs. (16) and (17), we obtain the following energy integral

$$\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2 + V(\psi, M) = 0 \tag{18}$$

where $V(\psi, M)$ is the Sagdeev potential and is given as

$$\begin{aligned} V(\psi, M) = & -\frac{1}{\left(1 - \frac{M^2}{n_i^3}\left(\alpha_c f e^{\alpha_c\psi} + \alpha_h(1-f)e^{\alpha_h\psi}\right) + \frac{5\sigma}{3n_i^{1/3}}\left(\alpha_c f e^{\alpha_c\psi} + \alpha_h(1-f)e^{\alpha_h\psi}\right)\right)^2} \\ & \times\left(-\frac{M^4}{2n_i^2}(1-n_i)^2 - M^2(1-\beta^2)\psi + M^2 H(\psi) + M^2\sigma\left(n_i^{5/3} - \frac{5}{2}n_i^{2/3} + \frac{3}{2}\right) + \beta^2\sigma H(\psi) + M^2\beta^2\sigma\left(\frac{1}{n_i} + \frac{3}{2}n_i^{2/3} - \frac{5}{2}\right)\right. \\ & \left. + \beta^2\sigma^2\left(n_i^{5/3} - \frac{1}{2}n_i^{10/3} - \frac{1}{2}\right) - \frac{\beta^2}{2}H(\psi)^2 - \frac{M^2\beta^2}{n_i}H(\psi) - \beta^2\sigma n_i^{5/3}H(\psi)\right) \end{aligned} \tag{19}$$

where

$$H(\psi) = \frac{f}{\alpha_c}(e^{\alpha_c\psi} - 1) + \frac{1-f}{\alpha_h}(e^{\alpha_h\psi} - 1) \tag{20}$$

In order to obtain the soliton solution from the energy integral Eq. (18), the Sagdeev potential has to satisfy the following conditions: $V(\psi, M) = 0$, $dV(\psi, M)/d\psi = 0$, $d^2V(\psi, M)/d\psi^2 < 0$ at $\psi = 0$; $V(\psi, M) = 0$ at some $\psi = \psi_m$, and $V(\psi, M) < 0$ for $0 < |\psi| < |\psi_m|$. For double layer solutions, an additional condition $dV(\psi, M)/d\psi = 0$ at $\psi = \psi_m$ (ψ_m is maximum amplitude) should be satisfied.

The condition $d^2V(\psi, M)/d\psi^2 < 0$ at $\psi = 0$ can be written as

$$\frac{d^2V(\psi, M)}{d\psi^2}\Big|_{\psi=0} = \frac{M^2 - M_0^2}{M^2 - M_1^2} < 0 \tag{21}$$

where

$$M_o^2 = \beta^2\left(\frac{1}{f\alpha_c + \alpha_h(1-f)} + \frac{5}{3}\sigma\right) = \beta^2\left(1 + \frac{5}{3}\sigma\right) \tag{22}$$

is the critical Mach number and

$$M_1^2 = \frac{1}{f\alpha_c + \alpha_h(1-f)} + \frac{5}{3}\sigma = 1 + \frac{5}{3}\sigma \tag{23}$$

since $f\alpha_c + \alpha_h(1-f) = 1$. From Eq. (21), further analysis shows that numerator $M^2 - M_o^2$ is always positive for soliton solution to exist as one must have $M > M_o$. Further more, in order to satisfy Eq. (21) the denominator $M^2 - M_1^2 < 0$, i.e., $M < M_1$. Therefore, M_1 is the upper limit of the Mach number beyond which solitary wave solutions will not exist. Thus, the low-frequency solitary wave structures in magnetized plasma may exist in the interval

$$M_o < M < M_1 \tag{24}$$

From Eq. (22) and (23), the condition (24) can be written as

$$\beta\sqrt{1 + \frac{5\sigma}{3}} < M < \sqrt{1 + \frac{5\sigma}{3}} \tag{25}$$

which gives the Mach number M values for fixed values of f, α_c and α_h. For the case of cold ions, i.e., $\sigma = 0$, Eqs. (19), (21)–(25) reduce to these derived by Rufai et al. [35]. In comparison to the earlier work, we observe that allowing for finite ion temperature ($\sigma \neq 0$) increases the upper limit of M beyond 1.

It is important to mention here that for the case of $\beta = 1 (\theta = 0)$ the parameter χ given by Eq. (16) becomes a constant. This is a singular case. Further, the normalizations used in the paper are such that we cannot recover the unmagnetized case by taking $\beta = 1$ in Eq. (18). In fact, for $\beta = 1 (\theta = 0)$, i.e., at parallel propagation, the inequality (25) cannot be satisfied because both critical Mach number (M_0) and upper limit (M_1) of the Mach number coincides with each other, thereby excluding the existence of soliton solutions. Therefore, our analysis is valid only for obliquely propagating weakly nonlinear ion-acoustic waves. This limitation of the model is probably due to the use of quasi-neutrality condition.

4. Numerical results

In this section, numerical computations of electrostatic potential ψ and Sagdeev potential $V(\psi, M)$ are carried out by solving Eqs. (18) and (19), respectively for the chosen set of parameters representative of the auroral region of the Earth's magnetosphere. The chosen plasma parameters are equilibrium cool and hot electron densities, N_{co}=0.2 cm^{-3}, and N_{ho}=1.8 cm^{-3}, cool and hot electron temperatures, T_c=1 eV, and T_h=26 eV which gives $T_{eff} \sim 7$ eV Berthomier et al. [28]. Firstly, in Fig. 1 we have shown the Mach number range (minimum Mach number M_o and maximum Mach number M_1) supported by the model for the existence of finite amplitude ion-acoustic solitons as a function of the ion temperature σ. We have plotted the these Mach numbers M_0 and M_1 obtained from Eqs. (22) and (23) for angle of propagation $\theta = 15^\circ$. It is interesting to note from Eq. (23) that maximum Mach number M_1 does not depend on the angle of propagation and that the supersonic solitons, i.e., with $M > 1$ can exist for finite value of σ. Further, The values obtained from analytical expressions (22) and (23) coincides with limits obtained through the numerical computations for the existence of the ion-acoustic solitons.

Fig. 2 shows the variation of the Sagdeev potential $V(\psi, M)$ with real potential ψ for different values of Mach number M as shown on the curves. The chosen fixed parameters are: cool to hot electron temperature ratio, $\tau = T_c/T_h$ = 0.04, normalized cool electron density, $f = n_{co}/n_o$ = 0.1, angle of propagation, $\theta = 35^\circ$ and normalized ion temperature, $\sigma = T_i/T_{eff}$ = 0.05. It is seen that the model supports negative potential solitons and their amplitude increases with increase in Mach number, M. It is interesting to note that double layer occurs at M=1.0365301. It is emphasized here that the nonlinear solitary structures

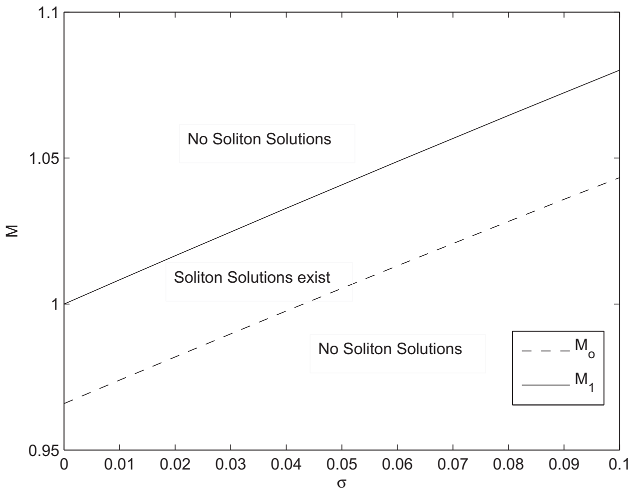

Fig. 1. Critical Mach number, M_0 and maximum Mach number, M_1 of ion-acoustic solitons shown as a function of the normalized ion temperature σ for $\theta = 15^\circ$.

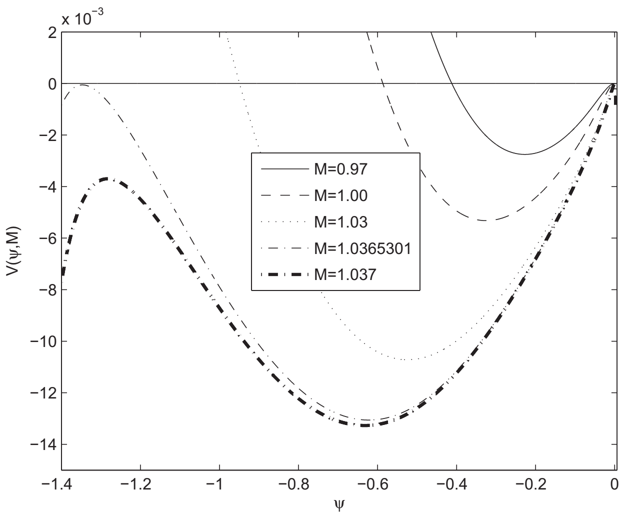

Fig. 2. Sagdeev potential, $V(\psi, M)$ vs normalized electrostatic potential ψ for various values of Mach number, M for τ= 0.04, f=0.1, σ=0.05, θ=35°.

are possible for subsonic ($M < 1$) as well as for supersonic ($M > 1$) for the current theoretical model whereas for the cold ion case, the soliton solutions are possible only for subsonic ($M < 1$) Mach number regime.

Fig. 3 shows the variation of Sagdeev potential versus the real potential ψ for different values of cool electron number density f and for other plasma parameters, $\tau = 0.04$, $\sigma = 0.01$, $\theta = 15^\circ$ and $M = 0.98$. The curves show that the solitary potential amplitude increases with an increase in cool electron density. For these parameters, the soliton solution are not possible beyond $f > 0.34$ whereas in the case of cold ions this limit is $f > 0.35$.

Fig. 4 shows the variation of Sagdeev potential $V(\psi, M)$ with real electrostatic potential ψ for different values of the cool to hot electron temperature ratio, $\tau = T_c/T_h$ for propagation angle $\theta = 15^\circ$ and other fixed parameters are the same as in Fig. 3. The ion-acoustic soliton amplitude increases with the increase in cool to hot electron temperature ratio. We also obtained the double layer solution which exist at cool to hot electron temperature ratio value $\tau = 0.092014$ whereas for cold ion case double layer solution is obtained at lower value of τ=0.0877117.

Fig. 5 shows the variation of Sagdeev potential $V(\psi, M)$ versus the normalized electrostatic potential ψ for different propagation angle, θ for f=0.1 and other fixed parameters of Fig. 4. The curves further show that as the angle of propagation θ increases, the soliton amplitude increases. It is very important to mention that at angle of propagation $\theta = 38.2885^\circ$ double layer structure appears and beyond $\theta > 38.2885^\circ$ there is no soliton or double layer solutions. It may be pointed out that for cold ion case the double layer appears at slightly lower angle of propagation, $\theta = 38.0425^\circ$ for the same set of parameters.

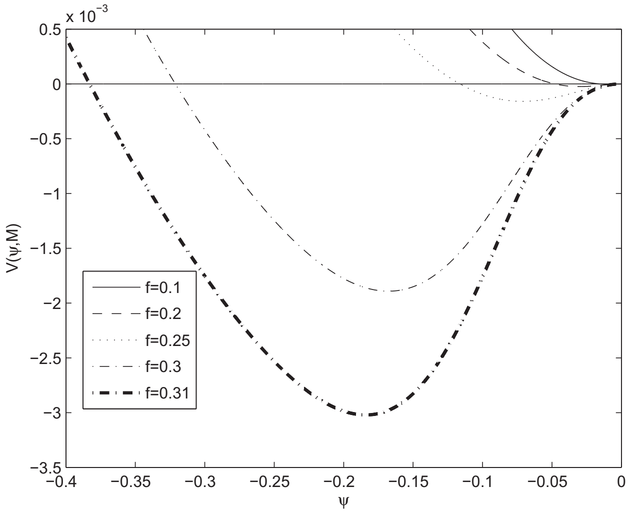

Fig. 3. Shows the variation of Sagdeev potential, $V(\psi, M)$ versus normalized potential, ψ for different values of cold electron density, f for the parameters, τ= 0.04, σ=0.01, θ=15° and M=0.98.

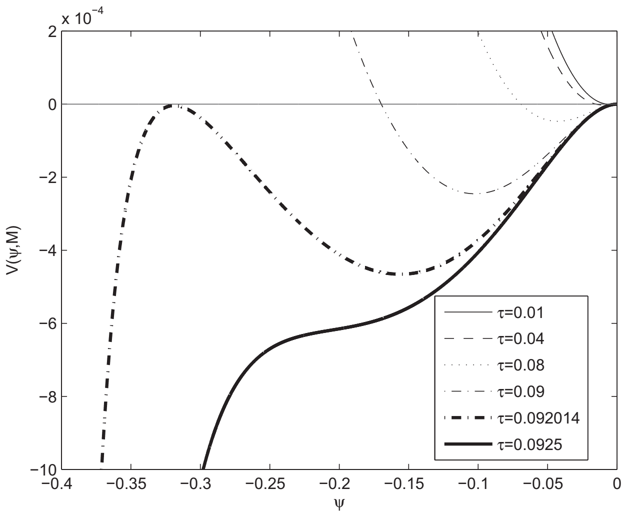

Fig. 4. Variation of Sagdeev potential, $V(\psi, M)$ with the normalized potential ψ for different values of cool to hot electron temperature ratio, τ for the parameters, θ= 15°, σ=0.01, f=0.1 and M=0.98.

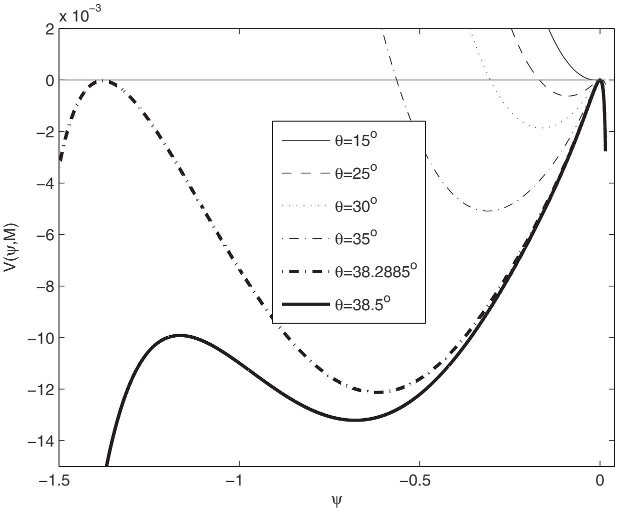

Fig. 5. Variation of Sagdeev potential, $V(\psi, M)$ with the normalized potential ψ, for τ= 0.04, σ=0.01, f=0.1 and M=0.98 for various values of angle of propagation, θ.

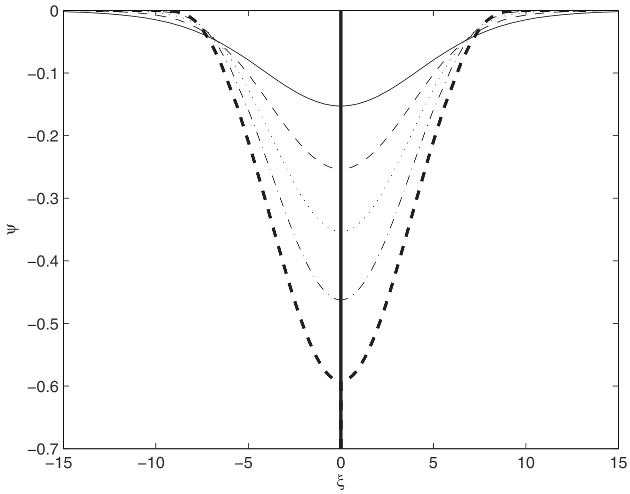

Fig. 6. Variation of electrostatic potential ψ vs ξ for τ= 0.04, θ=35° f=0.1, M=0.98, σ=0.2 (—), σ=0.15 (- - -), σ=0.1 (...), σ=0.05 (-.) and σ=0.0 (— — —).

Fig. 6 shows the variation of electrostatic potential ψ against ξ for ion-acoustic solitons for different values of σ for $\theta = 35^\circ$. The other fixed parameters are, $\tau = 0.04$, $f = 0.1$ and $M = 0.98$. It clearly shows that as the ion temperature increases, the ion-acoustic solitary potential amplitude as well as its width decreases. It is important to note that solitary potential amplitude reduces significantly with the inclusion of finite ion temperature. Here, $\sigma = 0$ corresponds to cold ion temperature[35].

Tables 1 and 2 describe the behaviour of the nonlinear structures for two different angles of propagation, $\theta = 20^0$ and 30^0, respectively with other parameters being $f = 0.1$, $\tau = 0.04$. It is clear from the Table 1 that the both minimum and maximum electric field amplitudes of the soliton increases with the increase in the ion temperature. This may be attributed to the fact that as ion temperature increases the minimum and maximum Mach numbers for which soliton solutions are obtained also increase, thus leading to an increase in electric field amplitude. The double layer solutions are not found for $\theta = 20^0$. The soliton width as well as pulse width decreases with the increase in ion temperature. For $\theta = 30^0$, the range of Mach number for solitons solutions widens as can be seen from the column 2 (Table 2) and can be verified by the condition (24). This leads to increases in electric field amplitudes (see column 4 of Table 2) with increases in ion temperature. At $\sigma = 0.19$ double layer solution appears. For double layers, the maximum electric field more or less remain constant. It is obvious from the two Tables that for $\sigma = 0$ (row 1, column 2 of Tables 1 and 2) solitary wave solutions are possible only for subsonic ($M < 1$) Mach numbers whereas for finite ion temperature (i.e., $\sigma \neq 0$) these solutions occur for both subsonic and supersonic ($M > 1$)Mach numbers.

Table 1
Properties of ion-acoustic solitons: Variation of Mach number range (M_o, M_1), Soliton speed, V (km/s), Electric field, E (mV/m), Soliton width, W (m) and Pulse duration, τ^* (ms), for various values of ion temperature ratio, $\sigma = T_i/T_{eff}$. Other chosen parameters are, angle of propagation, $\theta = 20^\circ$, Cool electron density, f = 0.1, cool to hot electron temperature ratio, τ = 0.04.

σ	M_0–M_1	V (km/s)	E (mV/m)	W (m)	τ^* (ms)
0.0	0.942 – 0.999	24.40 –25.9	0.082 – 5.57	4.98.68 – 176.8	20.44 – 6.83
0.01	0.95 – 1.008	24.61 – 26.11	0.095 – 5.705	484.12 – 176.8	19.67 – 6.11
0.04	0.975 – 1.0327	25.25 – 26.75	0.234 – 6.27	376.48 – 176.28	14.91 – 6.59
0.06	0.993 – 1.0488	25.72 – 27.16	0.498 – 6.71	305.76 – 174.72	11.89 – 6.43
0.08	1.01 – 1.0645	26.16 – 27.57	0.747 – 7.14	275.08 – 173.68	10.52 – 6.3
0.1	1.04 – 1.08	26.94 – 27.97	2.65 – 7.55	201.24 – 173.68	7.47 – 6.21
0.15	1.09 – 1.118	28.23 – 28.96	4.94 – 8.83	184.08 – 170.56	6.52 – 5.89
0.2	1.125 – 1.154	29.14 – 29.89	5.74 – 10.07	178.88 – 169.52	6.14 – 5.67

Table 2
Properties of ion–acoustic solitons and double layers: Variation of Mach number range (M_o, M_1), Soliton speed, V (km/s), Electric field, E (mV/m), Soliton width, W (m) and Pulse duration, τ^* (ms), for various values of ion temperature ratio, $\sigma = T_i/T_{eff}$. Other chosen parameters are, angle of propagation, $\theta = 30^\circ$, Cool electron density, f = 0.1, cool to hot electron temperature ratio, τ = 0.04. The double layer cases are identified as DL in column 2.

σ	M_o–M_1	V (km/s)	E (mV/m)	W (m)	τ^* (ms)
0.0	0.87 – 0.999	22.53 – 25.9	0.134 – 14.33	576.16 – 193.96	25.57 – 7.49
0.01	0.88 – 1.008	22.79 – 26.11	0.294 – 14.73	456.56 – 195	20.03 – 7.47
0.05	0.918 – 1.0408	23.79 – 26.96	1.14 – 17.02	312.26 – 195.52	13.13 – 7.25
0.1	0.96 – 1.08	24.86 – 27.97	2.17 – 20.52	270.4 – 198.12	10.88 – 7.083
0.15	0.992 – 1.118	25.69 – 28.96	2.46 – 25.58	260.0 – 205.4	10.12 – 7.09
0.19	1.02 – 1.142068 (DL)	26.42 – 29.69	3.03 – 25.96	254.8 – 335.4	9.64 – 11.3
0.2	1.03 – 1.149769973 (DL)	26.68 – 29.78	3.60 – 24.05	248.04 – 360.88	9.30 – 12.12
0.22	1.045 – 1.15688654 (DL)	27.07 – 29.96	4.13 – 24.28	243.67 – 353.08	9.00 – 11.79
0.24	1.058 – 1.1639855654 (DL)	27.40 – 30.15	4.45 – 24.36	241.28 – 347.36	8.81 – 11.52
0.26	1.07 – 1.1710685 (DL)	27.71 – 30.33	4.69 – 24.34	239.2 – 344.24	8.63 – 11.35
0.28	1.085 – 1.178133545463(DL)	28.10 – 30.51	5.40 – 24.29	234.0 – 343.2	8.33 – 11.25
0.3	1.098 – 1.185182322 (DL)	28.44 – 30.69	5.94 – 24.56	228.8 – 332.8	8.05 – 10.84

5. Discussion and conclusions

We have studied the obliquely propagating arbitrary amplitude ion-acoustic solitary waves and double layers in a magnetized auroral plasma consisting of two Maxwellian electrons and hot adiabatic ions. The model predicts negative potential solitons and double layers in the auroral region of the Earth's magnetosphere.

It is important to point out that inclusion of finite ion temperature pushes the limits of both minimum and maximum Mach numbers for solitons to higher sides, which results in higher range of electric field amplitudes. The new and surprising result which emerges from the current study is that for the case of hot ions, the nonlinear soliton/double layer structures are obtained for both subsonic ($M < 1$) and supersonic ($M > 1$) Mach number regimes, whereas for the case of cold ions only the subsonic ($M < 1$) solitary wave structures are possible. However, for a fixed Mach number, an increase in the ion temperature results in the reduction of the amplitude of the ion-acoustic solitons (cf. Fig. 6). It is also observed from numerical computations that for higher values of ion temperature ratio, the double layers occur at smaller propagation angle. For example, for $\sigma = 0.3$, the ion-acoustic double layers occur at $\theta = 27^0$ for Mach number, M=1.21936756 (not shown). The soliton electric field amplitude increases with increase in propagation angle θ, as seen from Tables 1 and 2 and Fig. 5.

The ion-acoustic solitary waves have been observed by several satellite missions throughout the different regions of the Earth's magnetosphere. The theoretical model developed here shows that amplitude of ion-acoustic solitons increases with Mach number, cool electron density and angle of propagation. The Viking satellite measurements in auroral zone reported electric field amplitude of $\sim$ (10–50) mV/m, spatial width of about 100 m, pulse width less than 20 ms and soliton speed (5–50) km/s with the plasma parameters N_{co}=0.2 cm^{-3}, N_{ho}=1.8 cm^{-3}, T_c=1 eV, T_h=26 eV which gives $T_{eff} \sim$ 7 eV Berthomier et al. [28]. For these observed parameters and σ=0.01, the maximum electric field amplitude for the solitons comes out to be in the range $\sim$ (0.3–15) mV/m (Refer to second row, column 4 of Table 2) for $\theta = 30^\circ$. The corresponding soliton width, pulse duration and speed come out to be $\sim$ (195–455) m, (7–20) ms and (22–26) km/s, respectively. For the ion-acoustic double layers, the electric field amplitude and soliton speed are higher whereas the width is smaller compared to the solitons. The present results are in fair agreement with the Viking satellite observations in auroral regions.

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