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Ion-acoustic supersolitons in the presence of non-thermal electrons

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ABSTRACT

A great deal of interest has been generated in the literature to study the newly found structures known as supersolitons. Theoretical models are being developed to understand these unusual electric field structures which have wiggle on their wings. The current understanding of these structures require at least three-component plasma model. In this work, ion-acoustic supersolitons are studied in three-component unmagnetized plasmas comprising of Boltzmann electrons, nonthermal (Cairn's distribution) electrons and fluid cold ions. The supersoliton solutions are obtained for Mach number values beyond the existence of double layers. The effect of nonthermality, cold electron density and cold to hot electron temperature ratio is examined on ion-acoustic supersolitons. It is found that presence of nonthermal electrons significantly affect the existence of the supersolitons. The amplitude of the supersolitons decreases with the increase in the nonthermality for the fixed value of Mach number. Our results show that electric field and electrostatic potential of the supersolitons are more distorted which are obtained for Mach numbers closer to the double layer solutions than the far away solutions. Due to the lack of observations of these structures, either in space or laboratory plasmas, our theoretical model predictions can not to be compared directly. However, it is worth investigating the data on the electrostatic solitary structures observed by several satellites in the various region of the Earth's magnetosphere.

1. Introduction

A number of authors in the past have studied ion-acoustic solitary waves in two temperature plasmas [1–6]. Reddy and Lakhina [7] studied small amplitude ion-acoustic solitons and double layers in auroral magnetized plasma consisting of hot and cold electrons and two ions (oxygen–hydrogen). They showed existence of either negative potential double layers, or negative potential solitons or positive potential solitons in distinct parametric space. They extended their work to include any number of ion beams and found that the fast and slow hydrogen (as well as oxygen) beam-acoustic modes can be generated [8]. It was shown that these nonlinear structures could be either rarefactive double layers or rarefactive or compressive solitons. The propagation characteristics of ion-acoustic rarefactive soliton in a two electron temperature unmagnetized plasma with two cold ions have been studied by Sayal et al. [9]. They used the kinetic and fluid models for both the ion species. Yadav et al. [10] have studied the nonlinear ion-acoustic periodic waves in two-electron temperature plasmas. Solitary waves and weak double layers in a two-electron temperature auroral plasma have been studied by Berthomier et al. [11] and applied

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their results to Viking satellite observations. Kourakis and Shukla [12] investigated ion-acoustic solitary waves in an unmagnetized, collisionless, three-component plasma, composed of positive cold ion fluid moving in a background of two thermalized electron population. Ghosh and Lakhina [13] studied the width-amplitude variation of rarefactive ion-acoustic solitary waves in the unmagnetized and magnetized multi-ion plasmas with two electrons temperature. The large amplitude ion acoustic solitons in unmagnetized multi-ion plasma with two electron temperatures have been studied by Verheest et al. [14].

Recently, since the discovery of unusual electric field structures known as supersolitons which have wiggles on their wings, lot of interest has been generated among the researchers to study these structures in various theoretical plasma models. The concept of supersolitons in plasma physics which is new, was introduced by Dubinov and Kolotkov [15,16] in multi-component plasmas. Initially, they could find supersolitons in five component plasmas but later on reported ion-acoustic supersolitons in four component plasmas with at least one of the electron species having Maxwellian distribution [17]. These supersolitons in plasmas arise when three consecutive local extrema of the Sagdeev pseudopotential between the undisturbed conditions and soliton amplitude are supported. The associated electric field of these structures is slightly distorted than the usual solitons. Though there is no report of direct observations of these structures in space plasmas, they may have already been observed in some of the satellite observations. Therefore, it is important to investigate carefully, the data on the electrostatic solitary structures observed in various region of the Earth's magnetosphere. Further, the signatures of supersolitons were already seen in the previous theoretical studies [18–21] but these structures have not been mentioned as 'supersolitons' in their works. For example, Baluku et al. [19] found that plasmas having two Boltzmann-distributed electrons and one fluid ion species can admit positive potential structures which now have been identified as supersolitons.

The lack of observations of these structures did not hold back the scientific community to study supersolitons in plasmas. Thus, recently some theoretical works have been done on the supersolitons in multi-component unmagnetized and magnetized plasmas including dusty plasmas. Verheest et al. [22] have studied electrostatic supersolitons in three species plasmas consisting of cold negative and positive fluid ions and nonthermal (Cairn's type) electrons. They have delineated the existence regimes of the supersolitons by determining the limiting factors. Ion-acoustic solitons have been studied in three-component unmagnetized plasmas comprising of cold fluid ions and two (cold and hot) electrons having kappa-distributions [23]. It was shown that the existence regime of the supersolitons vary significantly with the spectral index kappa. Dust ion acoustic supersolitons have been studied in the dusty plasmas with stationery negative dust, cold fluid protons and nonthermally distributed electrons [24]. The existence of large amplitude dust-acoustic solitons and supersolitons has been examined dusty plasmas comprising of cold negative dust, adiabatic positive dust, Boltzmann electrons, and nonthermal ions. The model supports the existence of positive potential supersolitons in addition to regular negative and positive potential solitons [25]. For the first time, Rufai et al. [26] reported obliquely propagating supersolitons in a magnetized three-component plasma model for certain range of plasma parameters. Recently, Lakhina et al. [27] have shown that supersolitons cannot exist in 3-component plasmas having two types of ion species and one type of electron species with Boltzmann distribution. The existence of ion acoustic supersolitons in two component plasmas has been ruled out by Verheest et al. [28]. Thus, the present understanding of supersolitons requires minimum of three-component plasmas.

The particle distributions in plasmas can very well depart from Maxwellian distributions and have nonthermal distributions. The nonthermal particle distributions such as Cairns distribution has been successfully used to model solitary waves in Earth's magnetosphere. The Cairns distribution can at best be described as Maxwellian distribution with an enhanced nonthermal tail [29]. They have used the nonthermal distribution to model both rarefactive and compressive solitary structures in the auroral region of the Earth's magnetosphere. Though, the solitary structures and double layers have been extensively studied in the literature using Cairns distribution, however, newly found nonlinear structures known as supersolitons have not been sufficiently investigated. In this paper, ion-acoustic supersolitons are studied in three-component unmagnetized nonthermal plasma. The theoretical model consists of Boltzmann electrons, nonthermal (Cairn's distribution) electrons [29] and fluid cold ions which has not yet been examined. In the next section theoretical model is presented and numerical results are discussed and concluded in Sections 3 and 4, respectively.

2. Theoretical model

We consider a homogeneous, unmagnetized three component plasma consisting of two types of electrons, namely, cold and hot electrons and fluid ions. The cold electrons are assumed to be having Maxwellian distribution and hot electrons follow the non-thermal distribution [29]

$$f_{0\ h}(v) = \frac{N_{0h}}{\sqrt{2\pi v_{th}^2}} \frac{\left(1 + \frac{\alpha v^4}{v_{th}^4}\right)}{(1+3\alpha)} exp\left(-\frac{v^2}{2v_{th}^2}\right) \tag{1}$$

where $N_{0h},\ v_{th} = \sqrt{T_h/m_e},\ m_e,\ T_h$ are the equilibrium density, thermal speed, mass and temperature of the hot electrons, respectively and α is a parameter which determines the population of energetic non-thermal electrons. The distribution of electrons in the presence of non-zero potential can be found by replacing $\frac{v^2}{v_{th}^2}$ by $\frac{v^2}{v_{th}^2} - \frac{2e\Phi}{T_h}$, where Φ is the electrostatic potential and e is the electronic charge. Thus, integration over the resulting distribution function gives the following expression for the hot electron density

$$n_h = (1-f)(1-\beta\phi+\beta\phi^2)exp(\phi), \tag{2}$$

where $\beta = \frac{4\alpha}{(1+3\alpha)}$ and the other governing equations of the model in the normalized form are given by

$$n_c = fexp(\phi/\tau) \tag{3}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{4}$$

$$\frac{\partial v_i}{\partial t} + v_i\frac{\partial v_i}{\partial x} + \frac{\partial \phi}{\partial x} = 0, \tag{5}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_c + n_h - n_i \tag{6}$$

where $f = N_{0c}/N_0;\ N_{0c}$ is the cold electron density at equilibrium and $N_0 = N_{0c} + N_{0h} = N_{0i}$ is the total electron (ion) density at equilibrium and $\tau = T_c/T_h$ is the cold to hot electron temperature ratio. We have normalized the densities by total electron density, N_0, velocities by acoustic speed electrons, $c_s = \sqrt{T_h/m_i}$, lengths by effective Debye length defined as $\lambda_{dh} = \sqrt{T_h/4\pi N_0 e^2}$, temperature by hot electron temperature T_h, time by inverse of ion plasma frequency $\omega_{pi}^{-1} = \sqrt{m_i/4\pi N_0 e^2}$, the potential by T_h/e, and, the thermal pressure by $N_0 T_h$.

To study the properties of arbitrary amplitude electrostatic ion-acoustic solitary waves, we transform the above set of Eqs. (2)–(6) to a stationary frame moving with velocity V, the phase velocity of the wave, i.e., $\xi = (x - M\,t)$, where $M = V/c_s$ is the Mach number (V is normalized with respect to the acoustic speed). Then, we solve for ion number density using Eqs. (4), (5) and obtain the following expression

$$n_i = \frac{M}{\sqrt{M^2 - 2\phi}} \tag{7}$$

and the Poisson Eq. (6) transforms to

$$\frac{\partial^2 \phi}{\partial \xi^2} = n_c + n_h - n_i \tag{8}$$

For ion density to be real valued, the above Eq. (7) defines the limitation on electrostatic potential ϕ, i.e., $\phi \leqslant M^2/2$. Substituting Eqs. (2), (3) and (7) into Poisson's Eq. (8) and assuming appropriate boundary conditions for the localized disturbances, i.e., $n_{c,h,i} \to f,\ (1-f),\ 1$ along with the conditions that $\phi = 0$, and $d\phi/d\xi = 0$ at $\xi \to \pm\infty$, we obtain the following energy integral,

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi, M) = 0, \tag{9}$$

where $V(\phi, M)$ is the Sagdeev potential given by

$$V(\phi, M) = M^2 - M\sqrt{M^2 - 2\phi} + f\tau[1 - exp(\phi/\tau)] + (1-f)\{1 + 3\beta - (1 + 3\beta - 3\beta\phi + \beta\phi^2)exp(\phi)\} \tag{10}$$

where $\tau = T_c/T_h;\ T_c$ & T_h are the temperatures of the cold and hot electrons, respectively.

The first two terms on the right hand side (r.h.s.) of Eq. (10) are the ion contribution, third and fourth terms are contributions from cold and hot electrons, respectively. In the absence of non-thermal electrons, i.e., for $\alpha = \beta = 0$, the fourth term reduces to usual Boltzmann distributed hot electron contribution.

Eq. (9) describes the motion of a pseudo particle of unit mass in a pseudo-potential $V(\phi, M)$. Here ϕ and ξ play the role of displacement from the equilibrium and time, respectively. In the case of solitons, the pseudo particle is reflected in the pseudo-potential field at some $\phi = \phi_0$ and returns to its initial state of $\phi = 0$ without any potential drop. However, for the double layers, due to an additional condition of charge neutrality at $\phi = \phi_0$ (ϕ_0 is the maximum amplitude), the pseudo particle cannot be reflected as both the pseudo-force and pseudo-velocity vanish there; instead it goes to another state with a net potential drop. Therefore, for soliton solutions of Eq. (9), the Sagdeev potential $V(\phi, M)$ must satisfy the following conditions: $V(\phi, M) = 0$, $dV(\phi, M)/d\phi = 0$, $d^2V(\phi, M)/d\phi^2 < 0$ at $\phi = 0$; $V(\phi, M) = 0$ at $\phi = \phi_0$ and $V(\phi, M) < 0$ for $0 < |\phi| < |\phi_0|$. For double layer solutions, an additional condition $dV(\phi, M)/d\phi = 0$ at $\phi = \phi_0$ has to be satisfied.

From Eq. (10) it can be seen that Sagdeev potential $V(\phi, M)$ and its first derivative with respect to ϕ vanish at $\phi = 0$. On the otherhand, the condition $d^2V(\phi, M)/d\phi^2 < 0$ at $\phi = 0$ is satisfied provided $M > M_0$, where M_0 is the critical Mach number which is given by

$$M_0 = \sqrt{\frac{\tau}{f + \tau(1-f)(1-\beta)}} \tag{11}$$

It is interesting to note that for Maxwellian distribution ($\alpha = 0 = \beta$) of electrons, our expression for minimum Mach number, M_0 given by Eq. (11) matches with the expression M_s, (normalized acoustic speed) in the limit $\kappa_c \to \infty, \kappa_h \to \infty$ given by Verheest et al. [24] (cf. their Eq. (7)). For soliton solution to exist, the Mach number, M should exceed M_0. Further, in our case the minimum Mach number, M_0 is higher compared to the case of Maxwellian electrons [19].

The upper M values for the regular positive potential ion-acoustic solitons occur either by the presence of double layer or infinite ion density compression. The positive potential ion-acoustic supersolitons occurs when the Sagdeev potential has three roots for finite ϕ or Sagdeev potential profile with respect to ϕ has three extrema or two potential well structures. There is no analytical method available for finding the existence domain of supersolitons. We would, therefore, rely on the numerical method based on Sagdeev potential profile.

3. Numerical results

In this section, numerical computations of electrostatic potential ϕ and Sagdeev potential $V(\phi, M)$ are carried out by solving Eqs. (9) and (10), respectively for the chosen set of parameters: $f = 0.01$, $\tau = 0.09$ and $\alpha = 0.01$ for various values of Mach number, M. These representative plasma parameters are taken from Baluku et al. [19] and Verheest et al. [24]. It can be seen from the graph (see Fig. 1) that the double layer solution appears at $M = 0.9944245$. The second well is not accessible for the Mach number corresponding to the double layer. It is interesting to see that for M exceeding the double layer Mach number ($M = 0.9944245$), the second well becomes accessible for the supersolitons to exist as seen from the graph for $M = 0.998$. It may be pointed out here that the shape of the two subwells appearing in the case of supersolitons are different than the one in the case of two temperature electrons having Maxwellian distributions (refer to Fig. 3(b) of [19] and Fig. 2, [24]). The first subwell (closer to $\phi = 0$) is very shallow whereas the second subwell (closer to $\phi = \phi_0$, where ϕ_0 is the maximum amplitude of the supersolitons for fixed set of parameters) is very deep in comparison to the subwells obtained in their models. Further, in our case the double layer occurs for $M/M_0 = 1.0251$ whereas for the Maxwellian distribution of electrons, the corresponding $M/M_0 = 1.0351$ [19,24]. The lower values of M/M_0 are expected as both minimum Mach number, M_0 as well as the Mach number, M for which double layers are found are increased with the presence of nonthermal hot electrons. Also, the amplitude of the double layers as well as of the supersolitons is higher.

In Fig. 2, we have plotted the Sagdeev potential $V(\phi, M)$ vs electrostatic potential ϕ for $f = 0.01$, $\alpha = 0.01$ for fixed value of Mach number, $M = 0.9982$ for different values of τ. The double layer occurs at $\tau = 0.0915$ and at lower value of τ supersoliton

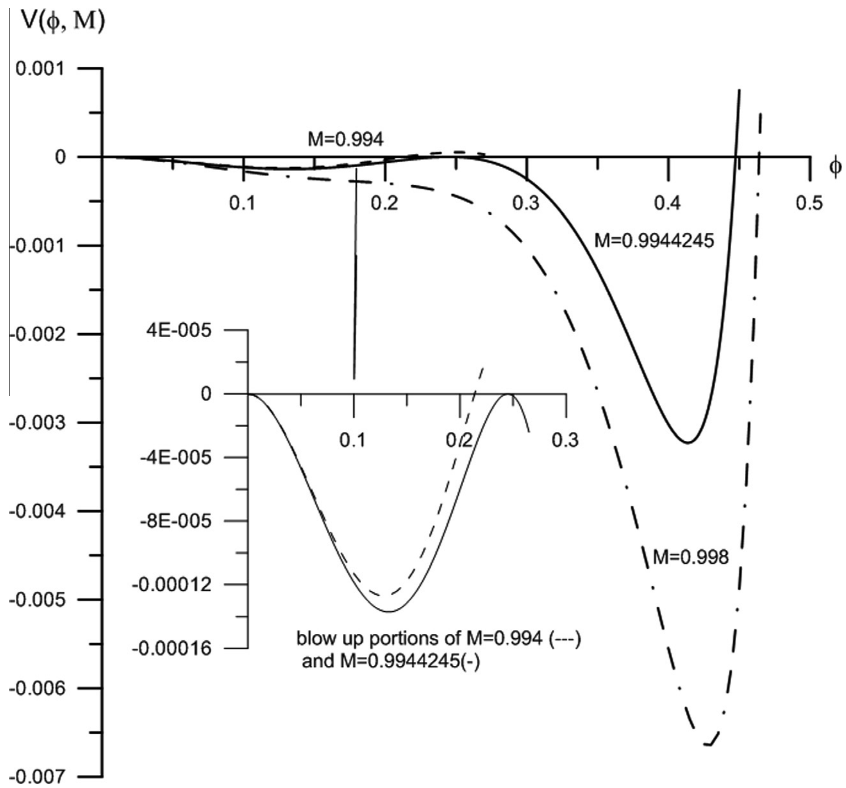

Fig. 1. Sagdeev potential, $V(\phi, M)$ versus normalized electrostatic potential ϕ for various values of Mach number, M for the parameters: Ratio of cold to hot electron temperatures, $\tau = T_c/T_h = 0.09$, normalized cold electron density, $f = N_{0c}/N_0 = 0.01$, nonthermal parameter $\alpha = 0.01$.

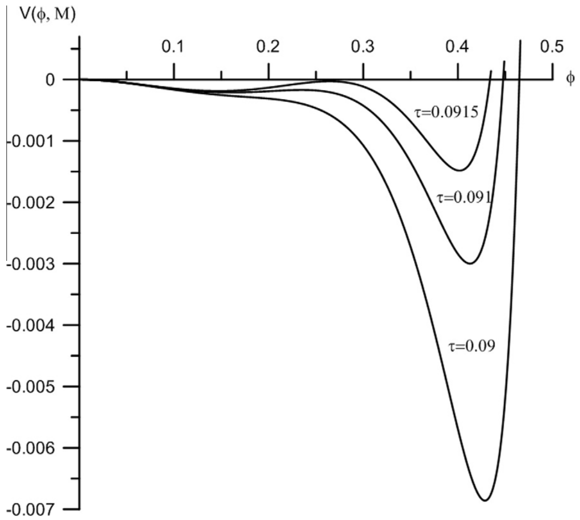

Fig. 2. Sagdeev potential, $V(\phi, M)$ versus normalized potential ϕ for various values of τ for $M = 0.9982$, $f = 0.01$, $\alpha = 0.01$.

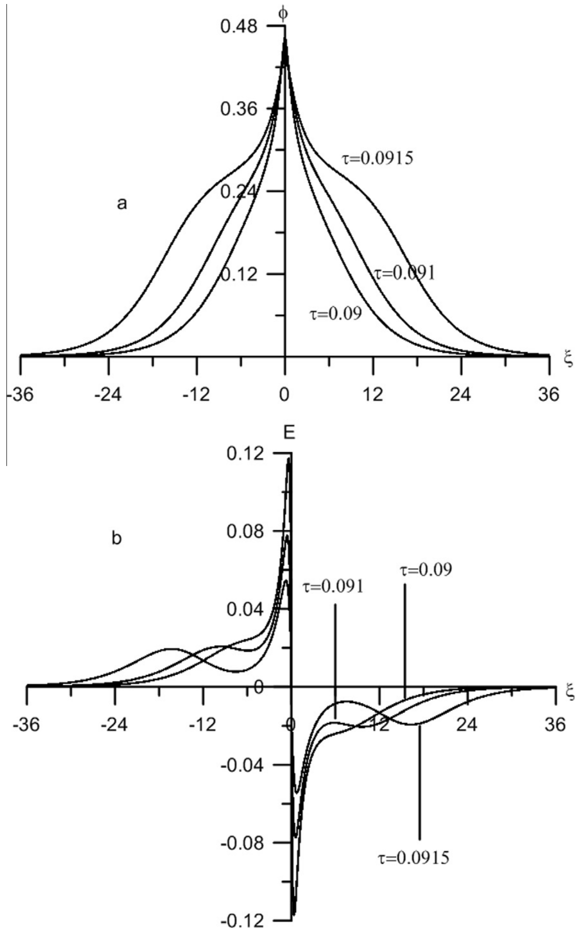

Fig. 3. (a) Electrostatic potential, ϕ versus ξ and (b) Electric field, E vs ξ for various values of $\tau = T_c/T_h$ as indicated on the curves for the Mach number, $M = 0.9982$. Other parameters are the same as in Fig. 1.

solutions are obtained. It is obvious from the figure that supersoliton amplitude decreases with the increase in the cold to hot electron temperature ratio. This feature is similar to the usual solitons in such plasmas. The depth of both the subwells also decreases with the increase in τ. The corresponding electrostatic potential and electric field profiles are shown in Fig. 3a and

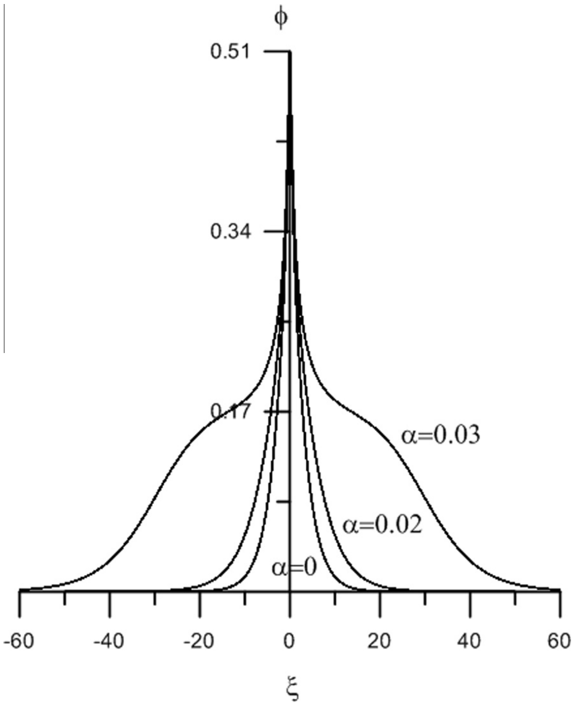

Fig. 4. Electrostatic potential, ϕ vs ξ for various values of α for M = 1.0157. Other parameters are the same as in Fig. 1.

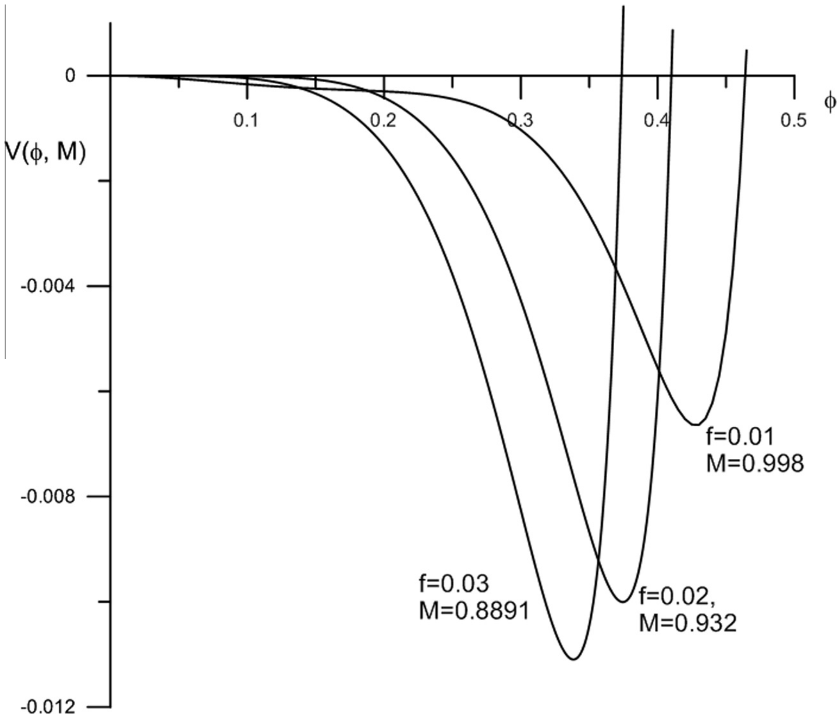

Fig. 5. Sagdeev potential, $V(\phi, M)$ versus normalized potential ϕ for $\tau = 0.09$ α = 0.01. Supersolitons are shown for a combination of different Mach number, M and cold electron density, f values.

b, respectively. These profiles have wiggles of a typical supersoliton and is more distorted for $\tau = 0.0915$ value. This implies that supersoliton solutions which are in the close vicinity of double layers have more distorted electric field or potential profiles than the far away supersolitons. It must be mentioned here that for the parameters of Fig. 2, the supersoliton amplitude limit $\phi \leqslant M^2/2$ comes out to be 0.4982. This exactly matches with the numerically obtained limit. Further, thorough numerical investigation of the existence regime reveals that supersolitons exist for $0.083 \leqslant \tau \leqslant 0.0915$ for the plasma parameters of Figs. 2 and 3. The maximum amplitude $\phi_{max} = 0.4982$ of the supersoliton is obtained for $\tau = 0.083$. Fig. 4 shows the electrostatic supersoliton potential, ϕ vs ξ for different values of the nonthermal parameter, α for $\tau = 0.09$, $f = 0.01$ and $M = 1.0157$. Here again the supersoliton amplitude decrease with the increase in nonthermality but width increases. It is observed that supersoliton do not exist beyond $\alpha > 0.03$ for the parameters of Fig. 4.

Fig. 5 shows the existence of supersolitons for various combinations of the cold electron density and Mach number. From the figure one can see that as cold electron density increases, the supersolitons occur at lower Mach numbers. This is also obvious from the critical Mach number expression given by Eq. (11). For the fixed values of τ and β, the critical Mach number decreases with increase in cold electron density. The amplitude of the supersolitons also decreases with increasing cold electron density. Numerical results also showed that the Mach number ranges for which supersoliton solutions are obtained narrows down with the increase in cold electron density. Further, first subwell becomes shallower whereas second subwell becomes deeper with the increase in the cold electron density.

4. Conclusions

Ion-acoustic supersolitons have been examined in three-component unmagnetized plasmas composed of Boltzmann electrons, nonthermal (Cairn's distribution) electrons and fluid cold ions. It is found that the presence of nonthermal electrons significantly affect the existence domains of the supersolitons. For the fixed value of the Mach number, the amplitude (width) of the supersolitons decreases (increases) with the increase in the nonthermality as well as cold to hot electron temperature ratio. In this model the respective shapes of the first (closer to $\phi = 0$) and second (closer to $\phi = \phi_0$) subwell are much shallower and deeper than the subwells obtained in a model with two Boltzmann electrons and fluid ions [19,24]. The Mach number ranges for which supersoliton solutions are obtained narrows down with the increase in cold electron density. It is obvious from the results that electric field and electrostatic potential of the supersolitons are more distorted which are obtained closer to the double layer solutions than the far away solutions.

In the literature, these novel structures known as supersolitons have not been directly reported in space or laboratory plasmas. Theoretical efforts are being made to understand these structures in multi-component unmagnetized and magnetized, space and dusty plasmas. However, it warrants a more closer investigation into the data on the electrostatic solitary structures observed by several satellites in the various region of the Earth's magnetosphere. It will not only validate the present theoretical models being studied but also provide different perspective to the understanding of the acceleration processes in the Earth's magnetosphere which are mainly based on soliton and double layers. Also, theoretical models can be further improved and refined by taking into account the real event observations.

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