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Influence of dust mass distributions on generalized Jeans–Buneman instabilities in dusty plasmas

Peter Meuris,[1] Frank Verheest[2] and G. S. Lakhina[3]

[1]Belgisch Instituut voor Ruimte-Aëronomie, Ringlaan 3, B-1180 Ukkel, Belgium
[2]Sterrenkundig Observatorium, Universiteit Gent, Krijgslaan 281, B-9000 Gent, Belgium
[3]Indian Institute of Geomagnetism, Dr Nanabhai Moos Marg, Colaba, Mumbai 400005, India

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Abstract. The linear dispersion law is given for a generalized Jeans–Buneman instability in the presence of a dust mass distribution. It is shown that this instability is easier to evoke when a reasonable power law mass distribution for the dust grains is assumed than for the mono-sized dust case. For Boltzmann distributed ions and electrons the dust-acoustic mode and the unstable Jeans mode are modified due to the self-gravitation and mass distribution. The frequency of the dust-acoustic mode is increased. For long wavelengths, a new stable mode occurs, due only to the dust mass distribution. The growth rate of the Jeans instability is increased, and the mode is extended beyond the usual wavelength domain. For high values of k a new instability arises, which is called the dust distribution instability. © 1997 Elsevier Science Ltd

1. Introduction

A dusty plasma consists of charged dust particles coupled to the ambient plasma through existing or self-induced electromagnetic fields. Depending upon the concentration, one has isolated screened dust grains (dust-in-plasma) or real dusty plasmas, where the charged dust participates in the Debye screening. Eminent reviews have been given by Goertz (1989), Northrop (1992), and Mendis and Rosenberg (1994). The study of dusty plasmas is a relatively new area of research both with respect to theoretical developments as well as specific applications. Recent interest in such plasmas has increased because of the observation of charged dust in the vicinity of comets and planets, and also in laboratory devices.

The analysis of waves and instabilities in dusty plasmas (recently reviewed by Shukla (1996) and Verheest (1996)) is interesting from a theoretical point of view in many aspects. Fluctuating dust charges and extra sink/source terms in the different equations, due to capture and emission of plasma particles, are two intriguing aspects. In this paper, however, we will look at a self-gravitating system, and investigate the influence of dust mass distributions on generalized Jeans–Buneman instabilities. These mass distributions are present in several astrophysical dusty plasmas (e.g. cometary environments, dusty planetary rings, spokes in Saturn's B-ring), over a broad range of sizes, and therefore we show that modelling these systems by a mono-size dust distribution must be carried out with care.

Jeans instability in dusty plasmas has been studied before (see, e.g. Bliokh and Yaroshenko, 1985; Pandey *et al.*, 1994; Avinash and Shukla, 1994; Verheest and Shukla, 1997). Pandey and Lakhina (1996) found that, in the presence of streaming, Jeans instability and the Buneman mode can overlap, and they called this mode for obvious reasons the Jeans–Buneman mode. We will see that the inclusion of a dust distribution leads to new modes or modifications of existing ones. In particular, for the usual micron-sized grains, the Jeans mode can be extended beyond the usual wavelength range, where it is called the dust distribution instability.

Our paper is structured as follows: in Section 2, the linear dispersion law is derived and then in Section 3, we look at the influence of the dust mass distribution on the dispersion law of the generalized Jeans–Buneman mode. For Boltzmann distributed ions and electrons, the Jeans instabilities are investigated in Section 4. Finally our conclusions are given in Section 5.

2. Fluid model equations and linear dispersion

We consider a plasma consisting of electrons, ions and charged dust grains, which may be immersed in a uniform external magnetic field directed along the z-axis. After

Correspondence to: F. Verheest

having recalled the fully general multispecies dispersion law, we will specialize our results to a dusty plasma in which the electrons (and sometimes also the ordinary plasma ions) are light enough so that we can neglect their inertial effects and describe the equilibrium densities by Boltzmann distributions. The dust grain species, on the other hand, are so massive that we may treat them as cold fluids. In between both extremes, the ions can display both temperature and streaming effects or alternatively, will also be treated as inertialess. We assume that the characteristic time of the instability is much smaller than the inverse of the charging frequency of the dust grains. Hence we can neglect source/sink terms in the different equations, as given by Bhatt and Pandey (1994).

Reverting to the general picture, the behaviour of electrostatic modes in the z-direction is governed by the following set of basic equations, which include for the different species the continuity equations

$$\frac{\partial n_\alpha}{\partial t}+\frac{\partial}{\partial z}(n_\alpha u_\alpha)=0 \tag{1}$$

together with the equations of motion

$$\frac{\partial n_\alpha}{\partial t}+u_\alpha\frac{\partial u_\alpha}{\partial z}=-\frac{1}{n_\alpha m_\alpha}\frac{\partial p_\alpha}{\partial z}-\frac{q_\alpha}{m_\alpha}\frac{\partial \phi}{\partial z}-\frac{\partial \psi}{\partial z}. \tag{2}$$

Here ϕ and ψ are the electrostatic and gravitational potentials, and the label α characterizes the different species making up the dusty plasma, each with density n_α, parallel fluid velocity u_α, charge q_α, mass m_α and scalar pressure p_α. For the latter we take a polytropic law as

$$p_\alpha \sim n_\alpha^{\gamma_\alpha}. \tag{3}$$

The description is closed by Poisson's equations, both for the electrostatic and gravitational effects

$$\varepsilon_0\frac{\partial^2\phi}{\partial z^2}+\sum_\alpha n_\alpha q_\alpha=0 \tag{4}$$

$$\frac{\partial^2\psi}{\partial z^2}=4\pi G\sum_\alpha n_\alpha m_\alpha. \tag{5}$$

Because of their high masses, one could be tempted to include only the contribution of the dust grains in equation (5), but that is an unnecessary restriction at this stage.

Following Bliokh and Yaroshenko (1985), we linearize and Fourier transform the relevant equations. This assumes that there exists some kind of equilibrium solution for the different variables. Because there is not such a thing as opposite masses, there is no way to make the gravitational potential disappear in the zeroth order, as is the case for the electrostatic potential. However, we might gain some important physical insight by invoking "Jeans swindle" and ignoring the zeroth order of the potential field of gravity in looking at local perturbations. We get the dispersion law as

$$\left(1-\sum_\alpha\frac{\omega_{p\alpha}^2}{(\omega-kU_\alpha)^2-k^2c_{s\alpha}^2}\right)\left(1+\sum_\alpha\frac{\omega_{J\alpha}^2}{(\omega-kU_\alpha)^2-k^2c_{s\alpha}^2}\right)+\left(\sum_\alpha\frac{\omega_{p\alpha}\omega_{J\alpha}}{(\omega-kU_\alpha)^2-k^2c_{s\alpha}^2}\right)^2=0. \tag{6}$$

We have introduced for the different species the plasma frequencies, defined through $\omega_{p\alpha}^2=N_\alpha q_\alpha^2/\varepsilon_0 m_\alpha$, the Jeans frequencies through $\omega_{J\alpha}^2=4\pi G N_\alpha m_\alpha$, and the thermal velocities through $c_{s\alpha}^2=\gamma_\alpha P_\alpha/N_\alpha m_\alpha$. Capital letters like N_α, P_α and U_α denote the corresponding equilibrium values, in this case the equilibrium densities, pressures and parallel streaming. Charge neutrality in equilibrium means for the usual electron–ion–dust models that

$$N_i=N_e+\sum_d Z_\alpha N_\alpha \tag{7}$$

and N_e is typically smaller than N_i for negatively charged dust grains with absolute charge numbers Z_α. In addition, we will work in the dust frame, and the lighter electrons will balance the ion current.

3. Jeans–Buneman instabilities

Before discussing equation (6) in more detail, we emphasize that if all species are cold ($c_{s\alpha}=0$) and not streaming ($U_\alpha=0$), then the last (squared) term vanishes exactly and there is a full decoupling between electrostatic and Jeans modes, regardless of the composition of the plasma (Bliokh et al., 1995). This is because $\omega_{p\alpha}\omega_{J\alpha}\sim N_\alpha q_\alpha$ includes the sign of the charges. In what follows we will, however, use the positive definitions for the different frequencies and write the signs of the charges explicitly.

We will specialize our results to a dusty plasma in which the dust is so massive that we may treat it as non-streaming, cold fluids. When we take into account that $\omega_{pi}^2\omega_{Je}^2\ll\omega_{pe}\omega_{pi}\omega_{Je}\omega_{Ji}\ll\omega_{pe}^2\omega_{Ji}^2$, and that $\omega_{Je}^2\ll\omega_{pe}^2$ and $\omega_{Ji}^2\ll\omega_{pi}^2$, this dispersion law becomes

$$\begin{aligned}&\left(1-\frac{\omega_{pe}^2}{\mathscr{L}_e}-\frac{\omega_{pi}^2}{\mathscr{L}_i}-\frac{1}{\omega^2}\sum_d\omega_{pd}^2\right)\left(1+\frac{1}{\omega^2}\sum_d\omega_{Jd}^2\right)\\&+\frac{1}{\omega^4}\left(\sum_d\omega_{pd}\omega_{Jd}\right)^2-\frac{\omega_{pe}^2\omega_{Ji}^2}{\mathscr{L}_e\mathscr{L}_i}-\left(\frac{\omega_{Je}^2}{\omega^2\mathscr{L}_e}+\frac{\omega_{Ji}^2}{\omega^2\mathscr{L}_i}\right)\\&\times\sum_d\omega_{pd}^2+2\left(\frac{\omega_{pe}\omega_{Je}}{\omega^2\mathscr{L}_e}-\frac{\omega_{pi}\omega_{Ji}}{\omega^2\mathscr{L}_i}\right)\sum_d\omega_{pd}\omega_{Jd}=0\end{aligned} \tag{8}$$

with for the plasma particles

$$\mathscr{L}_\alpha=(\omega-kU_\alpha)^2-k^2c_{s\alpha}^2. \tag{9}$$

Furthermore, we will prove that

$$\omega_{Je}^2\sum_d\omega_{pd}^2\ll\omega_{pe}^2\sum_d\omega_{Jd}^2,\quad \omega_{Ji}^2\sum_d\omega_{pd}^2\ll\omega_{pi}^2\sum_d\omega_{Jd}^2. \tag{10}$$

Indeed, this is the case for a mono-sized dust distribution, because

$$\frac{\omega_{J\alpha}^2}{\omega_{p\alpha}^2}\sim\frac{m_\alpha^2}{q_\alpha^2} \tag{11}$$

and in all known dusty plasmas the mass-per-charge ratio is much larger than for protons and normal ions, so that

$$\frac{m_e}{|q_e|}\ll\frac{m_d}{|q_d|},\quad\frac{m_i}{q_i}\ll\frac{m_d}{|q_d|}. \tag{12}$$

As we now show, a mass distribution for the different dust grains renders (10) easier to fulfil than in the mono-dust case.

For charged dust with radii a in a given range $[a_{\min}, a_{\max}]$, the differential density distribution goes like

$$n(a)\,\mathrm{d}a = Ka^{-\beta}\,\mathrm{d}a. \tag{13}$$

This kind of distribution is widely accepted in space plasmas. We find values of $\beta = 4.6$ for the F-ring of Saturn (Showalter *et al.*, 1992), while for the G-ring values of $\beta = 7$ and 6 were obtained by Gurnett *et al.* (1983) and Showalter and Cuzzi (1993). For cometary environments, we recall a value of $\beta = 3.4$ (McDonnell *et al.*, 1987).

When we assume that all grain sizes $a \ll \lambda_{\mathrm{De}}$, we can express mass and charge of a dust particle as follows:

$$m(a) = \frac{4}{3}\pi\rho a^3 \sim a^3 \tag{14}$$

$$q(a) = 4\pi\varepsilon_0 a V_0 \sim a \tag{15}$$

with ε_0 the vacuum permittivity, ρ the mass density of the grains (assumed to be constant and equal for all grains) and V_0 the electric surface potential at equilibrium (assumed to be constant as well).

To keep the discussion tractable, we start with two different dust species, with respective grain sizes a_1 and a_2, and densities $N_{\mathrm{d}1}$ and $N_{\mathrm{d}2}$. The average size is logically given by

$$\bar{a} = \frac{N_{\mathrm{d}1}a_1 + N_{\mathrm{d}2}a_2}{N_{\mathrm{d}1} + N_{\mathrm{d}2}}. \tag{16}$$

We thus can use (14) and (15) to define

$$\begin{aligned} m_{\mathrm{d}1} &= m(a_1),\ q_{\mathrm{d}1} = q(a_1) \\ m_{\mathrm{d}2} &= m(a_2),\ q_{\mathrm{d}2} = q(a_2) \\ \bar{m}_{\mathrm{d}} &= m(\bar{a}),\ \bar{q}_{\mathrm{d}} = q(\bar{a}) \end{aligned} \tag{17}$$

with corresponding notations for the individual and averaged dust grain plasma and Jeans frequencies. As there are more grains of smaller size than larger ones, we can introduce the following quantities:

$$\delta = \frac{a_1}{a_2} < 1, \quad \nu = \frac{N_{\mathrm{d}2}}{N_{\mathrm{d}1}} \le \delta < 1. \tag{18}$$

The hypothesis that $\nu \le \delta$ amounts to the observation that $N_{\mathrm{d}1}a_1 > N_{\mathrm{d}2}a_2$, in other words the densities fall steeper with radius than linear, as indicated already earlier. We need to investigate the following ratio:

$$\begin{aligned} R &= \frac{\sum_{\mathrm{d}}\omega_{\mathrm{Jd}}^2}{\sum_{\mathrm{d}}\omega_{\mathrm{pd}}^2} \Big/ \frac{\bar{\omega}_{\mathrm{Jd}}^2}{\bar{\omega}_{\mathrm{pd}}^2} \\ &= \frac{(N_{\mathrm{d}1}a_1^3 + N_{\mathrm{d}2}a_2^3)(N_{\mathrm{d}1}+N_{\mathrm{d}2})^4 a_1 a_2}{(N_{\mathrm{d}1}a_1 + N_{\mathrm{d}2}a_2)^4(N_{\mathrm{d}1}a_2 + N_{\mathrm{d}2}a_1)} \\ &= \frac{(\delta^3+\nu)(1+\nu)^4\delta}{(\delta+\nu)^4(1+\delta\nu)} \\ &= 1 + \frac{\nu(1-\delta)^2\{\delta^2(1-\delta)+\nu^2(2\delta+4\delta^2-\nu)+R'\}}{(\delta+\nu)^4(1+\delta\nu)} \end{aligned} \tag{19}$$

with

$$R' = \delta[1+\delta+\nu(2+\delta)(2+\nu^2)] > 0. \tag{20}$$

As can be seen this ratio is larger than one under the assumptions made. That means that explicit use of two dust species increase the ratio R, and assumption (10) is even easier to fulfil, as we have

$$\frac{\omega_{\mathrm{Ji}}^2}{\omega_{\mathrm{pi}}^2} \ll \frac{\bar{\omega}_{\mathrm{Jd}}^2}{\bar{\omega}_{\mathrm{pd}}^2} < \frac{\sum_{\mathrm{d}}\omega_{\mathrm{Jd}}^2}{\sum_{\mathrm{d}}\omega_{\mathrm{pd}}^2}. \tag{21}$$

Of course, the same reasoning *a fortiori* holds for three or more dust species, by repeating the type of argument whenever two dust species are replaced by their averages.

Along the same paths, we can prove that

$$\omega_{\mathrm{pe}}\omega_{\mathrm{Je}}\sum_{\mathrm{d}}\omega_{\mathrm{pd}}\omega_{\mathrm{Jd}} \ll \omega_{\mathrm{pe}}^2\sum_{\mathrm{d}}\omega_{\mathrm{Jd}}^2, \quad \omega_{\mathrm{pi}}\omega_{\mathrm{Ji}}\sum_{\mathrm{d}}\omega_{\mathrm{pd}}\omega_{\mathrm{Jd}} \ll \omega_{\mathrm{pi}}^2\sum_{\mathrm{d}}\omega_{\mathrm{Jd}}^2. \tag{22}$$

Indeed, for a mono-sized dust distribution, the proof is immediate. A similar reasoning holds for a power law dust distribution, where we now need to investigate the ratio

$$\begin{aligned} \hat{R} &= \frac{\sum_{\mathrm{d}}\omega_{\mathrm{Jd}}^2}{\sum_{\mathrm{d}}\omega_{\mathrm{pd}}\omega_{\mathrm{Jd}}} \Big/ \frac{\bar{\omega}_{\mathrm{Jd}}}{\bar{\omega}_{\mathrm{pd}}} \\ &= \frac{(N_{\mathrm{d}1}a_1^3 + N_{\mathrm{d}2}a_2^3)(N_{\mathrm{d}1}+N_{\mathrm{d}2})^2}{(N_{\mathrm{d}1}a_1 + N_{\mathrm{d}2}a_2)^3} \\ &= \frac{(\delta^3+\nu)(1+\nu)^2}{(\delta+\nu)^3} \\ &= 1 + \frac{\nu(1-\delta)^2\{1+2\delta+2\nu+\nu\delta\}}{(\delta+\nu)^3} \end{aligned} \tag{23}$$

which also turns out to be greater than 1. Hence the dispersion law (8) is to a high degree of precision approximated by

$$\left(1 - \frac{\omega_{\mathrm{pe}}^2}{\mathscr{L}_{\mathrm{e}}} - \frac{\omega_{\mathrm{pi}}^2}{\mathscr{L}_{\mathrm{i}}} - \frac{1}{\omega^2}\sum_{\mathrm{d}}\omega_{\mathrm{pd}}^2\right)\left(1 + \frac{1}{\omega^2}\sum_{\mathrm{d}}\omega_{\mathrm{Jd}}^2\right) + \frac{1}{\omega^4}\left(\sum_{\mathrm{d}}\omega_{\mathrm{pd}}\omega_{\mathrm{Jd}}\right)^2 - \frac{\omega_{\mathrm{pe}}^2\omega_{\mathrm{Ji}}^2}{\mathscr{L}_{\mathrm{e}}\mathscr{L}_{\mathrm{i}}} = 0. \tag{24}$$

Now we treat the electrons as an inertialess Boltzmann species. The condition for that is phase velocities much smaller than the electron thermal velocities, $\mathrm{Re}\,\omega/k \ll c_{\mathrm{se}}$. Introducing the electron Debye length as usually by $\lambda_{\mathrm{De}} = c_{\mathrm{se}}/\omega_{\mathrm{pe}}$, and defining $A = k^2\lambda_{\mathrm{De}}^2/(1+k^2\lambda_{\mathrm{De}}^2)$, we can rewrite the dispersion law as

$$\begin{aligned} &[(\omega - kU_{\mathrm{i}})^2 - k^2c_{\mathrm{si}}^2 - A\omega_{\mathrm{pi}}^2][\omega^2 + \omega_{\mathrm{Jd}}^2 - A\omega_{\mathrm{pd}}^2] \\ &\quad = A^2\omega_{\mathrm{pi}}^2\omega_{\mathrm{pd}}^2 + \frac{A}{2\omega^2}[(\omega - kU_{\mathrm{i}})^2 - k^2c_{\mathrm{si}}^2] \\ &\quad\quad \times \sum_{\mathrm{d}}\sum_{\mathrm{d}}(\omega_{\mathrm{pd}}\omega_{\mathrm{Jd}} - \omega_{\mathrm{pd}}\omega_{\mathrm{Jd}})^2 - \frac{\omega^2\omega_{\mathrm{Ji}}^2}{1+k^2\lambda_{\mathrm{De}}^2}. \end{aligned} \tag{25}$$

We have used the total dust plasma and Jeans frequencies over the different dust grain species, by putting

$$\omega_{\rm pd}^2 = \sum_{\rm d} \omega_{\rm p\alpha}^2, \quad \omega_{\rm Jd}^2 = \sum_{\rm d} \omega_{\rm J\alpha}^2. \tag{26}$$

Thus our results generalize recent derivations of Pandey and Lakhina (1996) by including both the electrons (albeit through their inertialess Boltzmann description), by allowing a variety of dust grains to be present, and by retaining a term in $\omega_{\rm Ji}^2$, which cannot always be neglected without proper justification.

If all the dust is of the same size and we assume that $\omega_{\rm Ji}^2 \ll A\omega_{\rm pi}^2$, the generalized Jeans–Buneman instabilities follow from

$$[(\omega - kU_{\rm i})^2 - k^2c_{\rm si}^2 - A\omega_{\rm pi}^2][\omega^2 + \omega_{\rm Jd}^2 - A\omega_{\rm pd}^2] = A^2\omega_{\rm pi}^2\omega_{\rm pd}^2. \tag{27}$$

If we put $A = 1$, thereby omitting the electron effects, our results include the Jeans–Buneman mode as discussed by Pandey and Lakhina (1996) in their equation (25). One of their possibilities arises in the regime where

$$\omega_{\rm Jd}^2 \simeq A\omega_{\rm pd}^2 \tag{28}$$

and the mode is unstable when

$$(\omega - kU_{\rm i})^2 - k^2c_{\rm si}^2 < A\omega_{\rm pi}^2. \tag{29}$$

From equation (27) we find for large wavelengths ($k \to 0$) the Jeans instability, while for small wavelengths ($k \to \infty$), without gravitational effects, the Buneman mode is recovered (Stix, 1992). As stated earlier, these two modes can overlap.

The influence of a dust mass distribution will alter the right-hand side of equation (25), but as long as it remains positive, the instability criterion remains formally the same. However, the possible regime is now determined by

$$\sum_{\rm d} \omega_{\rm J\alpha}^2 \simeq A \sum_{\rm d} \omega_{\rm p\alpha}^2 \tag{30}$$

and a mass distribution for the different dust grains renders this easier to fulfil than in the mono-dust case. One can also prove that explicit use of more dust species increases the total plasma and Jeans frequencies compared to the corresponding frequencies computed from averaged quantities, but in such a way, as we saw already, that the Jeans frequencies increase faster, rendering the fulfilment of the criterion (28) easier.

4. Jeans instabilities

Here we take the complementary point of view from the preceding section, and suppose that both the electrons and the (positive) ions can be treated as inertialess Boltzmann species. The condition is now that phase velocities are much smaller than the electron and ion thermal velocities, $\mathrm{Re}\,\omega/k \ll c_{\rm si}, c_{\rm se}$. Introducing also the ion Debye length by $\lambda_{\rm Di} = c_{\rm si}/\omega_{\rm pi}$, and redefining A as standing for

$$A = \frac{k^2\lambda_{\rm D}^2}{1 + k^2\lambda_{\rm D}^2} \tag{31}$$

with $1/\lambda_{\rm D}^2 = 1/\lambda_{\rm De}^2 + 1/\lambda_{\rm Di}^2$, we find from the dispersion law (6) without additional approximations a simple biquadratic equation:

$$\omega^4 + (\omega_{\rm Jd}^2 - A\omega_{\rm pd}^2)\omega^2 - \frac{A}{2}\sum_{\rm d}\sum_{\rm d'}(\omega_{\rm pd}\omega_{\rm Jd'} - \omega_{\rm pd'}\omega_{\rm Jd})^2 = 0. \tag{32}$$

As the product of the roots of equation (32) for ω^2 is negative, we always have a real solution for ω^2, corresponding to a stable mode, as well as a purely imaginary mode ($\omega^2 < 0$). The roots are given by

$$\frac{\omega^2}{\omega_{\rm pd}^2} = \frac{1}{2}\left(A - B \pm \sqrt{(A-B)^2 + 2AC}\right) \tag{33}$$

with

$$B = \frac{\omega_{\rm Jd}^2}{\omega_{\rm pd}^2} \tag{34}$$

$$C = \frac{1}{\omega_{\rm pd}^4}\sum_{\rm d}\sum_{\rm d'}(\omega_{\rm pd}\omega_{\rm Jd'} - \omega_{\rm pd'}\omega_{\rm Jd})^2. \tag{35}$$

Both coefficients would be very small for typical micron-sized dust grains, and it is only for highly charged millimetre-sized dust that $B \simeq 1$, a condition used by Pandey and Lakhina (1996) without further comments for the Jeans–Buneman instability following from equation (27). For the discussion of the modes we remark that

$$2B - C = \frac{2}{\omega_{\rm pd}^4}\left(\sum_{\rm d}\omega_{\rm pd}\omega_{\rm Jd}\right)^2 > 0 \tag{36}$$

so that we find simple bounds for the discriminant in equation (33) from

$$(A-B)^2 \le (A-B)^2 + 2AC = (A+B)^2 - 2A(2B-C) < (A+B)^2. \tag{37}$$

In addition, we get from the special case $A = B < 1$ a critical wave number given by

$$k_{\rm c} = \frac{1}{\lambda_{\rm D}}\sqrt{\frac{\omega_{\rm Jd}^2}{\omega_{\rm pd}^2 - \omega_{\rm Jd}^2}} \tag{38}$$

and can introduce the dust-acoustic frequency $\omega_{\rm da}$ by

$$\omega_{\rm da}^2 = A\omega_{\rm pd}^2 = \frac{k^2\lambda_{\rm D}^2\omega_{\rm pd}^2}{1 + k^2\lambda_{\rm D}^2}. \tag{39}$$

The description of the modes goes as follows (see also Table 1):

- When $A > B$ or $\omega_{\rm da} > \omega_{\rm Jd}$, the plasma effects dominate, with a permissible k range determined by $k_{\rm c} < k$. In the solution of equation (33) the square root with the plus sign gives a generalized dust-acoustic wave (Rao *et al.*, 1990), at a higher frequency due to the dust distribution.

  The minus sign in front of the square root gives rise to an unstable mode, the frequency of which vanishes when C and the dust mass distribution go to zero. Hence we will call this new mode a *dust distribution instability*.
- In the special case that $A = B$ or $k = k_{\rm c}$, the dust distribution leads to stable and unstable modes, with

$$\omega^2 = \pm\,\omega_{\rm pd}\omega_{\rm Jd}\sqrt{\frac{C}{2}}. \tag{40}$$

Table 1. Overview of the different modes included in equation (32)

		Stable modes $\omega^2 > 0$	Unstable modes $\omega^2 < 0$
$B < 1$	$k < k_c$	New stable mode $0 \le \omega^2 < \omega_{da}^2$	Jean's instability $-\omega_{Jd}^2 < \omega^2 \le \omega_{da}^2 - \omega_{Jd}^2 < 0$
	$k = k_c$	New stable mode $\omega^2 = \omega_{pd}\omega_{Jd}\sqrt{C/2}$	Dust distribution instability $\omega^2 = -\omega_{pd}\omega_{Jd}\sqrt{C/2}$
	$k_c < k$	Dust-acoustic mode $0 < \omega_{da}^2 - \omega_{Jd}^2 \le \omega^2 < \omega_{da}^2$	Dust distribution instability $-\omega_{Jd}^2 < \omega^2 \le 0$
$B \ge 1$	all k	New stable mode $0 \le \omega^2 < \omega_{da}^2$	Jeans instability $-\omega_{Jd}^2 < \omega^2 \le \omega_{da}^2 - \omega_{Jd}^2 < 0$

Without dust distribution these are all zero-frequency modes.

- For the case $A < B$ the self-gravitational effects dominate, and the square root with the plus sign gives a new stable mode, which does not exist without dust mass distribution. For the limits on k we have to distinguish between the possibilities that $A < 1$, with then $k < k_c$, whereas for $A < 1 \le B$ there are no restrictions on k. This latter case is highly unlikely for the usually considered micron-sized grains.

  The mode with the minus sign in front of the square root is a modified Jeans mode, and the dust mass distribution increases its growth rate.

In all fairness it has to be remarked that as for the usually considered micron-sized dust grains both B and C are small, there is only a very narrow k-range (with correspondingly very large structures) for both the new stable mode as well as for the Jeans instability.

5. Conclusions

Although in most of the literature on the physics of dusty plasmas a mono-sized dust distribution is used, we expect in real life to find a dust size distribution. We examined the effect of such a distribution on the Jeans–Buneman mode for a self-gravitating dusty plasma. A power law distribution is used, widely accepted in several space plasma applications. We showed that the Jeans–Buneman instability is easier to evoke when such a mass distribution for the dust grains is assumed than for the mono-sized dust case.

For Boltzmann distributed ions and electrons, we see that the dust-acoustic mode and the unstable Jeans mode are modified, due to the self-gravitation and mass distribution. The frequency of the dust-acoustic mode is increased. For long wavelengths, a new stable mode occurs, due only to the dust mass distribution. The mass distribution will increase the growth rate of the Jeans instability. For high values of k a new instability arises, which we have called the dust distribution instability.

Given the modifications in the description of Jeans–Buneman modes induced by dust mass distributions, it seems likely that other wave modes in dusty plasmas will also be affected, and hence subsequent investigations into these new features are highly warranted.

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