OBLIQUE SOLITARY ALFVÉN MODES IN RELATIVISTIC ELECTRON-POSITRON PLASMAS

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Abstract. In electron-positron plasmas the charge-to-mass ratio is the same for both species. This leads for different waves to the vanishing of certain coefficients in the dispersion laws and nonlinear evolution equations, and also to the decoupling of some of the plasma modes. In particular, there is a low-frequency mode which exists at all angles of propagation with respect to the static magnetic field, corresponding at parallel propagation to a degenerate case of circularly polarized waves, and at perpendicular propagation to part of the extraordinary mode. The nonlinear evolution of this generalized X–mode is governed by a Korteweg–de Vries equation, valid at all angles of propagation except strictly parallel propagation, for which a different approach had been given already. The nonlinearity is strongest at perpendicular propagation.

1. Introduction

Large amplitude low-frequency electromagnetic waves in relativistic plasmas are believed to play an important role in many astrophysical problems, for example the slowing down of pulsars, pulsar radiation, acceleration of cosmic rays, extragalactic radio sources, double jets, etc. (Rees, 1971; Asseo, 1984), as well as in the studies of laser-pellet interactions, free-electron lasers and in ionospheric modification experiments using high-power radars (Shukla *et al.*, 1985). The oblique rotator model of pulsars predicts the possible existence of large amplitude low-frequency waves beyond the light cylinder of the pulsars (Pacini, 1968; Goldreich and Julian, 1969; Ostriker and Gunn, 1969; Rees, 1971). Lakhina and Tsintsadze (1990) have shown that the ultrarelativistic plasma in the wave zone of the pulsar can support two types of electromagnetic modes having frequencies close to the rotation frequency of the pulsar. These large amplitude low-frequency waves can accelerate electrons to relativistic energies and at the same time carry away rotational angular momentum and energy of the pulsars, thus leading to the slowing down of pulsars (Kennel *et al.*, 1979; Rees, 1971).

According to Ruderman and Sutherland (1975) the pulsar magnetosphere is dominated by a relativistic electron-positron plasma, although in general all three species, namely, electrons, positrons and ions could be present. Kennel and Pellat (1976) have shown that when the ions are driven relativistically by large amplitude waves, electrons and ions make equal contributions to the dispersion relation. Therefore, when the wave energy density greatly exceeds the rest mass energy

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density, it is justified to neglect differences in rest mass energy between ions and electrons, and such relativistic plasmas can be simply described as electron-positron plasmas with $m_i = m_e$. One may recall that the usual electron-ion plasmas can have many different time and spatial scale lengths, because of the difference in mass between the electrons and the ions. Most of these scales do not exist in electron-positron plasmas, and any cooperative phenomenon which originates from the mass difference between the electron and the ion could not manifest itself in electron-positron plasmas, *e.g.*, ion-acoustic waves, lower hybrid waves, etc.

Electromagnetic waves in relativistic plasmas pertaining to magnetospheres of pulsars and active galactic nuclei have been investigated rather intensively in the literature (Sturrock, 1971; Max and Perkins, 1972; Max, 1973; Kennel and Pellat, 1976; Buti, 1978; Sweeney and Stewart, 1978; Lakhina and Buti, 1981; Asseo, 1984; Lominadze *et al.*, 1983; Shukla, 1985; Stenflo *et al.*, 1985; Shukla *et al.*, 1986; Lakhina and Tsintsadze, 1990; Shukla and Stenflo, 1993). Large amplitude Alfvén waves propagating parallel to the external magnetic field in relativistic electron-positron have been studied by Sakai and Kawata (1980a,b), Mikhailovskii *et al.* (1985a,b,c) and Verheest (1996). The selfconsistent analysis of Verheest (1996) showed that the evolution of nonlinear parallel Alfvén modes is described by a vector form of a modified Korteweg–de Vries (mKdV) equation. Since large-amplitude, low-frequency modes are most suited for the acceleration of charged particles to very high energies (Kennel and Pellat, 1976; Asseo, 1984; Lakhina and Tsintsadze, 1990), it is of practical interest to investigate also the nonlinear evolution of large amplitude obliquely propagating modes. In this paper, we have studied large amplitude Alfvén modes propagating at an arbitrary angle to the external magnetic field in a relativistic electron-positron plasma by employing a reductive perturbation technique.

Our paper differs from previous treatments in important aspects. We have not neglected the displacement current, nor imposed quasi-neutrality, because in pulsars the magnetic field can be large, so that both the gyrofrequencies and the Alfvén velocity can be comparable to the plasma frequencies or the speed of light, respectively. Also, we have not imposed a particular polarization for the waves under study, although the low-frequency approach inherent in the reductive perturbation technique will automatically pick out the generalized extraordinary (X) mode which we discuss below. We have moreover let the formalism decide how the dependent variables are to be expanded order by order. The results of this way of dealing with the problem indicate that nature itself imposes quasi-neutrality for low-frequency phenomena, and that the usual Korteweg–de Vries (KdV) scaling emerges naturally.

The paper is organized as follows. In Section 2 we delineate the theoretical model and discuss what linear modes are possible in a magnetized electron-positron plasma at arbitrary angles of propagation compared to the direction of the static field. It turns out that in this plasma the low-frequency generalized X–mode decouples from the other modes, at all angles. Section 3 then gives the nonlinear evolution of that mode, although the required polarization is not imposed *a priori*. The result is a standard KdV equation, as also found in ordinary plasmas, even though in such plasmas the X–mode is mixed with other waves. Finally, our conclusions are given in Section 4.

2. Theoretical model and linear modes

For reasons of mathematical tractability our model is that of a homogeneous, cold plasma immersed in a uniform magnetic field, and composed of equal numbers of electrons and positrons. We will look at waves or structures propagating along the z–axis, so that all quantities depend only on z and t. For oblique modes we take $\mathbf{B}_0 = B_0\mathbf{e}_B$, with $\mathbf{e}_B = \sin\vartheta\mathbf{e}_x + \cos\vartheta\mathbf{e}_z$ the unit vector along the static magnetic field. The basic fluid equations (Shukla *et al.*, 1986) include the continuity equations,

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial z}(n_\alpha u_{\alpha z}) = 0, \tag{1}$$

together with the equations of motion,

$$\frac{\partial \mathbf{P}_\alpha}{\partial t} + u_{\alpha z}\frac{\partial \mathbf{P}_\alpha}{\partial z} = \mp e(\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}). \tag{2}$$

$\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, and the label α characterises the electrons ($\alpha = e$, with charge $q_e = -e$ and upper signs in the equations) and the positrons ($\alpha = p$, with $q_p = e$ and lower signs), with densities n_α, velocities $\mathbf{u}_\alpha$ and momenta $\mathbf{P}_\alpha$. The latter are defined in their relativistic form as

$$\mathbf{P}_\alpha = \frac{m\mathbf{u}_\alpha}{\sqrt{1 - \dfrac{u_\alpha^2}{c^2}}}, \tag{3}$$

with $m = m_e = m_p$ the rest mass. The description is closed by Maxwell's equations

$$\mathbf{e}_z \times \frac{\partial \mathbf{E}}{\partial z} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0}, \tag{4}$$

$$c^2\mathbf{e}_z \times \frac{\partial \mathbf{B}}{\partial z} = \frac{\partial \mathbf{E}}{\partial t} + \frac{e}{\varepsilon_0}(n_p\mathbf{u}_p - n_e\mathbf{u}_e), \tag{5}$$

$$\frac{\partial E_z}{\partial z} = \frac{e}{\varepsilon_0}(n_p - n_e). \tag{6}$$

Before going to the nonlinear development, we look at linear wave propagation, by linearizing the relevant equations (1)–(6) and Fourier transforming them, which yields the wave equation

$$c^2k^2(\omega^2 - \Omega^2)E_z\mathbf{e}_z + \omega_p^2\Omega^2\mathbf{E}\cdot\mathbf{e}_B\mathbf{e}_B + \left[(\omega^2 - c^2k^2)(\omega^2 - \Omega^2) - \omega_p^2\omega^2\right]\mathbf{E} = \mathbf{0}. \tag{7}$$

The total plasma frequency ω_p is defined through $\omega_p^2 = 2Ne^2/\varepsilon_0 m$, with $N = N_e = N_p$ the common equilibrium density, and $\Omega = eB_0/m$ is the gyrofrequency in absolute value, for both species. There is thus at all angles of wave propagation a decoupling of E_y, the component of $\mathbf{E}$ perpendicular to the plane containing $\mathbf{k}$ and $\mathbf{B}_0$, with dispersion law

$$\omega^4 - \omega^2(c^2k^2 + \omega_p^2 + \Omega^2) + c^2k^2\Omega^2 = 0, \tag{8}$$

from the other components. This mode corresponds at parallel propagation to a degenerate case of circularly polarized waves, and at perpendicular propagation to part of the X–mode. We hence will call it a generalized X–mode, existing at all angles of wave propagation. Now (8) has a high-frequency as well as a low-frequency branch, and in the latter, long-wavelength limit the linear phase velocity goes as (Sakai and Kawata, 1980a,b; Shukla *et al.*, 1986; Verheest, 1996)

$$\frac{\omega}{k} = V_A - \frac{V_A^5\omega_p^2}{2\Omega^4c^2}k^2 + \cdots \tag{9}$$

The Alfvén velocity V_A is given through

$$V_A^2 = \frac{c^2\Omega^2}{\omega_p^2 + \Omega^2}, \tag{10}$$

and reduces in the limit of $\Omega^2 \ll \omega_p^2$ to the familiar expression

$$V_A^2 \simeq \frac{c^2\Omega^2}{\omega_p^2} = \frac{B_0^2}{2\mu_0 Nm}, \tag{11}$$

with a factor 2 as the proper limit in an electron-positron plasma. However, in pulsars the magnetic field can be large, and hence modifications can occur in V_A if $\omega_p \sim \Omega$. This is also the reason why we cannot *a priori* neglect the displacement current in Ampère's law (5), nor deviations from charge neutrality in Poisson's law (6), as usual in a low-frequency approach.

The other components of $\mathbf{E}$, those in the plane spanned by $\mathbf{k}$ and $\mathbf{B}_0$, obey a rather complicated dispersion law, which we shall not discuss further, except to indicate what happens in the limiting cases of parallel or perpendicular propagation. For parallel propagation ($\mathbf{B}_0 \parallel \mathbf{e}_z$) there occurs a further decoupling between E_x and E_z, where E_x has the same dispersion as E_y, given by (8), while E_z corresponds to the cold-plasma analogue of plasma oscillations, at $\omega = \omega_p$. For perpendicular propagation ($\mathbf{B}_0 \parallel \mathbf{e}_x$) one of the waves corresponds to the ordinary (O) mode, having the usual dispersion law $\omega^2 = c^2k^2 + \omega_p^2$, whereas the other one is a fixed-frequency pure upper-hybrid mode with $\omega^2 = \omega_p^2 + \Omega^2$, the other remnant of the X–mode. Recall that in ordinary plasmas the X–mode cannot be factorized, and that for oblique propagation all modes are mixed together.

3. Nonlinear evolution

While for parallel low-frequency waves in ordinary plasmas the phase velocity has a linear correction in k (Mio *et al.*, 1976), we find in (9) a quadratic correction for the oblique modes, which interestingly enough is also the case for strictly parallel modes in electron-positron plasmas. This influences the stretching, which has to be taken as

$$\xi = \varepsilon(z - V_A t), \qquad \tau = \varepsilon^3 t, \tag{12}$$

As in the parallel case (Verheest, 1996), we will let the equations determine how the dependent variables are expanded consistently and start for all variables from

$$f = F + \varepsilon f_1 + \varepsilon^2 f_2 + \cdots. \tag{13}$$

All equilibrium quantities like F are zero, except for the densities and the components of the static field, as indicated already. An expansion in ε is for many reasons a more prudent way of attacking the problem, because in pulsars the magnetic field can be large, the masses are equal and it is not obvious how the different variables scale. Substitution of the stretching (12) and the perturbation expansions (13) into the equations (1)–(6) gives a sequence of equations, upon equating the coefficients of the various powers of ε.

As can be checked by going through the straightforward but time-consuming algebra to even higher order than needed to derive the nonlinear evolution equation (as we did, although the details will not be given here for reasons of brevity), the generalized X–mode is such that to all orders the electron and positron quantities obey

$$\begin{aligned} n &= n_e = n_p, \\ u_x &= u_{ex} = u_{px}, \\ u_y &= -u_{ey} = u_{py}, \\ u_z &= u_{ez} = u_{pz}. \end{aligned} \tag{14}$$

This means that we can restrict ourselves to the positron (or the electron) quantities from the outset, as will be done in what follows for the sake of the presentation. Analogous results hold for the wave fields, with

$$\begin{aligned} &E_x = E_z = 0, \\ &B_y = 0, \qquad B_z = B_{\|0}. \end{aligned} \tag{15}$$

Hence we need only consider the following set of fluid equations,

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(n u_z) = 0,$$
$$\left(\frac{\partial}{\partial t} + u_z \frac{\partial}{\partial z}\right) P_x = e B_{\|0} u_y,$$

$$\left(\frac{\partial}{\partial t}+u_z\frac{\partial}{\partial z}\right)P_y = e(E_y+u_zB_x-u_xB_{\|0}),$$
$$\left(\frac{\partial}{\partial t}+u_z\frac{\partial}{\partial z}\right)P_z = -eB_xu_y, \tag{16}$$

together with the nonzero Maxwell's equations

$$\frac{\partial E_y}{\partial z}=\frac{\partial B_x}{\partial t},$$
$$c^2\frac{\partial B_x}{\partial z}=\frac{\partial E_y}{\partial t}+\frac{2en}{\varepsilon_0}u_y. \tag{17}$$

This X–mode is characterized by zero currents and electric fields in the plane containing the static magnetic field and the direction of propagation, and also by strict quasi-neutrality. None of these characteristics were assumed or imposed at the outset, but emerged naturally as a consequence of the low-frequency approach.

Two different combinations can be formed from the x– and z–components of the equations of motion in (16), namely an equation for u_y,

$$u_y=\frac{B_{\|0}}{eB^2}\left(\frac{\partial}{\partial t}+u_z\frac{\partial}{\partial z}\right)P_x-\frac{B_x}{eB^2}\left(\frac{\partial}{\partial t}+u_z\frac{\partial}{\partial z}\right)P_z, \tag{18}$$

and an additional relation between P_x and P_z,

$$B_x\left(\frac{\partial}{\partial t}+u_z\frac{\partial}{\partial z}\right)P_x+B_{\|0}\left(\frac{\partial}{\partial t}+u_z\frac{\partial}{\partial z}\right)P_z=0. \tag{19}$$

The square of the total magnetic field is defined as $B^2=B_x^2+B_{\|0}^2$, and for the static part $B_0^2=B_{\perp0}^2+B_{\|0}^2$. Equation (18) can be used for the different orders in u_y, needed finally in Ampère's law in (17). The lower orders for P_x (or for u_x) and P_z (or for u_z) needed for that can be found from the y–component of the equation of motion in (16), together with (19). The continuity equation and the x–component of the equation of motion are only required for the two lowest orders in ε.

If we now were to start from nonzero E_{y1} and B_{x1}, we would find to lowest nonzero order that

$$n_1 = \frac{NB_{\perp0}}{B_0^2}B_{x1},$$
$$u_{x1} = -\frac{V_AB_{\|0}}{B_0^2}B_{x1},$$
$$u_{z1} = \frac{V_AB_{\perp0}}{B_0^2}B_{x1},$$
$$u_{y2} = \frac{V_A^2}{\Omega B_0}\frac{\partial B_{x1}}{\partial\xi}. \tag{20}$$

That is enough to let Ampère's law vanish to second order, due to the definition of V_A in (10), whereas after some more straightforward algebra a bifurcation point is reached at third order, in the sense that

$$B_{x1} B_0 \sin\vartheta = 0. \tag{21}$$

This means that for really oblique propagation ($\vartheta \neq 0$ and $\sin\vartheta$ finite), we need $B_{x1} = 0$, and all first-order variables vanish. The expansion thus starts at order ε^2 in our scheme, whereas for strictly parallel propagation it starts at order ε and leads to a vector equivalent of the modified Korteweg–de Vries (mKdV) equation (Verheest, 1996).

Now to lowest nonzero order we notice that

$$\begin{aligned}
n_2 &= \frac{N B_{\perp 0}}{B_0^2} B_{x2}, \\
u_{x2} &= -\frac{V_A B_{\parallel 0}}{B_0^2} B_{x2}, \\
u_{z2} &= \frac{V_A B_{\perp 0}}{B_0^2} B_{x2}, \\
u_{y3} &= \frac{V_A^2}{\Omega B_0} \frac{\partial B_{x2}}{\partial \xi}, \\
u_{y4} &= \frac{V_A^2}{\Omega B_0} \frac{\partial B_{x3}}{\partial \xi},
\end{aligned} \tag{22}$$

and Ampère's law vanishes to third and fourth orders in ε. Going further through the algebra to order ε^5, one deduces a KdV equation

$$\frac{1}{V_A}\left(1 + \frac{\Omega^2}{\omega_p^2}\right)\frac{\partial B_{x2}}{\partial \tau} + \frac{3\sin\vartheta}{2B_0} B_{x2} \frac{\partial B_{x2}}{\partial \xi} + \frac{V_A^2}{2\Omega^2}\frac{\partial^3 B_{x2}}{\partial \xi^3} = 0. \tag{23}$$

As a KdV equation, this is related to what is known for oblique modes in a standard hydrogen plasma (Kakutani *et al.*, 1968) in the following way. When adapting their treatment to equal mass particles the coefficient of the dispersive term becomes positive definite and independent of the angle between the directions of wave propagation and external magnetic field, due to the fact that, whereas in hydrogen plasmas the electron and ion gyrofrequencies are vastly different, some exact cancellations now occur. These are also responsible for the decoupling of some of the linear modes, which normally are coupled in ordinary plasmas, as we pointed out already in the introduction and in §2.

Furthermore, the definition of the Alfvén velocity and of the (total) plasma frequency is different. Moreover, in view of possible applications to pulsars, it is more prudent, as we have done here, to check that the usual assumptions concerning low-frequency modes are obeyed by an electron-positron plasma. In particular, whereas quasi-neutrality indeed follows to the orders looked at, the displacement current has a non negligible influence, both on the definition of the Alfvén velocity V_A and of the coefficient of the time derivative in the KdV equation (23).

At perpendicular propagation our results tally with recent work on modes in two-ion plasmas (Toida and Ohsawa, 1994), again in the proper limits of single ion (positron) species and in the absence of the displacement current.

We can easily write (23) in dimensionless form with standard coefficients by the substitution

$$B_{x2} \to B_0 b, \qquad \xi \to \frac{V_A\sqrt{2}}{\Omega}\,\xi', \qquad \tau \to \frac{4\sqrt{2}}{\Omega}\left(1+\frac{\Omega^2}{\omega_p^2}\right)\tau', \tag{24}$$

and get, omitting the primes on the independent variables,

$$\frac{\partial b}{\partial \tau} + 6\sin\vartheta\, b\frac{\partial b}{\partial \xi} + \frac{\partial^3 b}{\partial \xi^3} = 0. \tag{25}$$

The remarkable property is that the nonlinearity is strongest at strictly perpendicular propagation, and vanishes if one tries to take the parallel limit. This latter property again shows that the case of strictly or very nearly parallel propagation has to be considered separately, leading to a completely different type of nonlinear behaviour (Verheest, 1996). The standard one-soliton solution of (23) is of the typical type

$$b = \frac{M}{2\sin\vartheta}\mathrm{sech}^2\left(\frac{\sqrt{M}}{2}(\xi - M\tau)\right). \tag{26}$$

4. Conclusions

In electron-positron plasmas, due to the equal charge-to-mass ratios between the negative and the positive charges, the mixing of the different timescales leads to substantial modifications of the linear and nonlinear evolution of waves. For low-frequency, long-wavelength Alfvén waves propagating at an angle to the external magnetic field and using a proper and consistent reductive perturbation analysis, we find a KdV equation. It has the remarkable property that the nonlinearity vanishes if one tries to take the parallel limit, and is strongest at strictly perpendicular propagation. For strictly parallel propagation, the expansion scheme starts at order ε, rather than at order ε^2 as in our scheme, and it leads to a vector equivalent of the modified Korteweg–de Vries (mKdV) equation (Verheest, 1996).

The nonlinear, obliquely propagating Alfvén solitons studied here would be relevant for some problems of pulsar physics, namely, the microstructure in the pulsar radiation or the subpulses, and the acceleration of charged particles to high energies. It is supposed that the subpulses are due to modulation by low-frequency Alfvén solitons of the high-frequency pulsar radiation (itself probably produced by the synchrotron radiation mechanism involving bunches of electrons/positrons moving along the magnetic field lines). Furthermore, interaction of Alfvén solitons with electrons may give rise to strong diffusion and scattering, leading to heating and acceleration of the latter to cosmic energies.

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