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Arbitrary amplitude solitary waves in plasmas with dust grains of opposite polarity and non-thermal ions

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Abstract. The existence of large amplitude solitary waves in a plasma comprised of a cold negative dust fluid, adiabatic positive dust fluid, Boltzmann electrons and non-thermal ions is theoretically investigated. Different regions in parameter space that correspond to different values of the ratio of the charge-to-mass ratios of the positive and negative dust grains have been identified where either negative or positive potential solitary wave structures occur and a region where coexistence of negative and positive potential solitary waves is supported.

1. Introduction

The field of dusty plasmas has attracted widespread interest from the start of the past decade due to the occurrence of massive, charged dust particles in various astrophysical and space plasma environments, such as planetary rings, dust rings of Phobos, cometary comae and tails, and in interplanetary space [1–5]. Because the dust is much more massive than the ions of typical ion-electron plasmas, a dusty plasma supports the propagation of ultra low-frequency waves such as the dust-acoustic wave (DAW), in which the massive dust provides the inertia and the restoring force arises from the pressures of the inertialess ions and electrons. The existence of the DAW was theoretically predicted by Rao et al. [6], and has been subsequently observed in a number of laboratory experiments [7–10].

Space plasma observations often indicate the existence of ion and electron populations that are not in thermodynamic equilibrium. They are best described by velocity distribution functions that have non-Maxwellian suprathermal tails. Non-thermal ion populations have been observed in the Earth's bow shock [11], Martian ionosphere [12] and near the Moon [13]. Non-thermal ion populations best modelled by a kappa-distribution have been found to occur in the magnetospheres of Jupiter and Saturn [14, 15]. Very recently, measurements by Cassini of thermal and suprathermal electron populations in the magnetosphere of Saturn are found to be well fitted by kappa-distributions [16].

The effect of ion and/or electron non-thermality on the existence of large amplitude solitary waves and/or double layers has been considered by a number of authors. Cairns et al. [17] considered a model comprising a cold ion fluid and non-thermal electrons. They found that non-thermal effects for the electrons is required for the occurrence of negative potential solitary waves (which coexist with positive potential structures) which could possibly account for the observations of electrostatic structures with density depletions by the Freja satellite [18]. Moving down to the regime of very low wave frequencies and phase speeds (in comparison with the electron and ion thermal speeds) associated with the dynamics of heavy dust, Mamun et al. [19] found that the coexistence of dust-acoustic solitary wave structures of negative and positive potential occurs for a plasma composed of negative dust and non-thermal ions. This study was followed by an investigation of linear and nonlinear dust-acoustic waves by Singh et al. [20] for an unmagnetized plasma comprising negative dust, Boltzmann electrons and non-thermal ions in which the effects of streaming dust was also considered. The findings of the study by Maharaj et al. [21] indicate that non-thermal effects of the ions is required for the occurrence of positive potential solitary waves that coexist with negative potential solitary structures for a dusty plasma also composed of negative adiabatic dust and Boltzmann electrons which is an extension of the adiabatic negative dust and non-thermal ion model considered by Mendoza-Briceño et al. [22]. Positive potential double layers studied in detail in [23] were found to limit the existence domain of the positive solitary structures from the high-Mach-number end, although it was not explicitly mentioned in [21] that a positive potential double layer restricts the existence domains of the positive potential solitary waves. Existence domains of large amplitude solitary structures are presented in [24] for a plasma composed of negative dust grains, non-thermal ions and Boltzmann electrons, whereas, solitary wave existence domains for a plasma with positive dust, non-thermal electrons and Boltzmann ions appear in [25]. The non-ideal corrections associated with the intermolecular cohesive forces and volume reduction effects have been included for the massive, negative dust grains in a study of large amplitude solitary structures for a non-thermal plasma [26]. The focus of the study by Verheest [27] was on small amplitude dust-acoustic solitons for a plasma comprised of any number of cold negatively charged drifting dust species, Boltzmann distributed hot and cool electrons and Boltzmann ions. The findings of the study by Chow et al. [28] indicate that secondary electron emission yields can be quite substantial for the smaller dust grains (due to a smaller distance that secondary electrons have to travel to reach the grain surface) resulting in positively charged dust grains that can coexist with larger negatively charged dust grains. Many authors have conducted studies based on two-dust plasma models taking into account the disparity in the masses of the oppositely charged dust species. The effect of non-thermal velocity distributions for the inertialess ions and electrons was investigated by Verheest [29] for a two-dust model comprised of cold positive dust, cold negative dust, non-thermal electrons and non-thermal ions, however, with the number densities of the non-thermal electrons and non-thermal ions reducing to the Boltzmann form with both species having the same temperature T if non-thermal electron and ion effects are ignored. Existence domains of negative potential solitary waves were found to be limited from the high-Mach-number end either by infinite negative dust compression or by the existence of a negative potential double layer. The occurrence of positive solitary waves was found to be limited from the high-Mach-number end by infinite positive dust compression, alone, because no positive potential double-layer solutions were found.

Mamun [30] considered a four-component plasma model comprised of cold negative dust, adiabatic positive dust, Boltzmann electrons and Boltzmann ions to investigate the existence of small and large amplitude solitary waves. The coexistence of large amplitude solitary waves of negative and positive potential was found to occur in some regions of parameter space. Upper limiting values of the range of Mach numbers for which solitary waves occur were not determined in this study. We consider a four-component plasma model comprising non-thermal ions, Boltzmann electrons, warm positive dust and cold negative dust which generalizes on the model considered in [30] because we include non-thermal effects of the ions. Our model reverts to that considered in [30] by simply setting $\gamma = 0$ to recover a Boltzmann distribution for the ions where γ is a measure of the deviation of the ions from a thermal Maxwellian velocity distribution. As in [30], we include dust pressure effects for only the positive dust grains but not for the negative ones which may be justified in view of the findings in [28] that the lighter (hence more mobile) dust grains are usually positively charged. In this paper, we identify the parameter regimes where large amplitude solitary waves occur. In Sec. 2, we present the model and the relevant equations. Our results and discussion appears in Sec. 3. Conclusions are presented in Sec. 4.

2. Model and governing equations

We consider a four component unmagnetized dusty plasma comprising a negatively charged cold dust fluid, a positively charged warm dust fluid, non-thermal ions and Boltzmann electrons. The ions are assumed to have a velocity distribution similar to that used to describe non-thermal electrons by Cairns et al. in [17] with the non-thermal ion number density given by $n_i = n_{i0}(1 + \beta\Psi + \beta\Psi^2)e^{-\Psi}$, where Ψ is the normalized electrostatic potential (with respect to $k_B T_i/e$) and $\beta = 4\gamma/(1+3\gamma)$ with γ being the ion non-thermal parameter [17]. For $\gamma = 0$, the ion number density reverts to the case of Boltzmann ions considered in [30]. The normalized number density for the Boltzmann electrons is given by $n_e = n_{e0}e^{\sigma\Psi}$, where $\sigma = T_i/T_e$. The basic set of equations in normalized form is given by

$$\frac{\partial n_{\mathrm{dn}}}{\partial t} + \frac{\partial(n_{\mathrm{dn}} v_{\mathrm{dn}})}{\partial x} = 0, \tag{2.1}$$

$$\frac{\partial v_{\mathrm{dn}}}{\partial t} + v_{\mathrm{dn}}\frac{\partial v_{\mathrm{dn}}}{\partial x} = \frac{\partial \Psi}{\partial x}, \tag{2.2}$$

$$\frac{\partial n_{\mathrm{dp}}}{\partial t} + \frac{\partial(n_{\mathrm{dp}} v_{\mathrm{dp}})}{\partial x} = 0, \tag{2.3}$$

$$\frac{\partial v_{\mathrm{dp}}}{\partial t} + v_{\mathrm{dp}}\frac{\partial v_{\mathrm{dp}}}{\partial x} = -\alpha\left(\frac{\partial \Psi}{\partial x} + \frac{\sigma_{\mathrm{dp}}}{n_{\mathrm{dp}}}\frac{\partial P_{\mathrm{dp}}}{\partial x}\right), \tag{2.4}$$

$$\frac{\partial P_{\mathrm{dp}}}{\partial t} + v_{\mathrm{dp}}\frac{\partial P_{\mathrm{dp}}}{\partial x} + 3P_{\mathrm{dp}}\frac{\partial v_{\mathrm{dp}}}{\partial x} = 0, \tag{2.5}$$

$$\frac{\partial^2 \Psi}{\partial x^2} = n_{dn} - \mu_{dp}n_{dp} + \mu_e e^{\sigma\Psi} - \mu_i(1 + \beta\Psi + \beta\Psi^2)e^{-\Psi}. \tag{2.6}$$

We have made use of the same normalizations as in [30]. The dimensionless quantity n_{dn} (n_{dp}) denotes the number density of the negative (positive) dust having been

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normalized by its equilibrium number density n_{dn0} (n_{dp0}), the negative (positive) dust fluid velocity in dimensionless form is denoted by v_{dn} (v_{dp}) having been normalized by $C_{dn} = \sqrt{Z_{dn}k_BT_i/m_{dn}}$, the electrostatic wave potential in dimensionless form is denoted by Ψ having been normalized by k_BT_i/e, the dimensionless pressure of the positive dust fluid is denoted by P_{dp} having been normalized by $n_{dp0}k_BT_{dp}$, where T_{dp} denotes the equilibrium temperature of the positive dust fluid. The ratio of the charge–mass ratios of the positive and negative dust species is given by $\alpha = Z_{dp}m_{dn}/(Z_{dn}m_{dp})$, $\sigma_{dp} = T_{dp}/(Z_{dp}T_i)$ is the normalized temperature of the positive dust, $\mu_e = n_{e0}/(Z_{dn}n_{dn0})$, $\mu_i = n_{i0}/(Z_{dn}n_{dn0})$ and $\mu_{dp} = Z_{dp}n_{dp0}/(Z_{dn}n_{dn0})$ are the normalized electron, ion and positive dust number densities, and $\sigma = T_i/T_e$. The equilibrium charge neutrality condition is assumed, viz. $\mu_i + \mu_{dp} = \mu_e + 1$, which takes into account the division of positive charge between the positive dust and the free positive charge. In the definitions above, $m_{dn}(m_{dp})$ is the mass of the negative (positive) dust grains, and $Z_{dn}(Z_{dp})$ denotes the number of electrons (positive charges) residing on a negative (positive) dust grain. The temperature of the ions (electrons) is denoted by T_i (T_e), e denotes the magnitude of the electronic charge and k_B is the Boltzmann constant. Time t is normalized with respect to ${\omega_{pn}}^{-1} = (m_{dn}/4\pi {Z_{dn}}^2 n_{dn0}e^2)^{1/2}$ and spatial variable is normalized by $\lambda_D = (Z_{dn}k_BT_i/4\pi {Z_{dn}}^2 n_{dn0}e^2)^{1/2}$.

Following the procedure in [19] and [22] to obtain solitary wave solutions, we make all variables in (2.1)–(2.6) a function of the co-moving coordinate ξ where $\xi = x - Mt$ (where the Mach number M is the speed of the nonlinear structure normalized with respect to the dust-acoustic speed C_{dn}) and finally obtain the following expressions for the number densities of the cold negative dust and adiabatic positive dust

$$n_{dn} = \frac{1}{\sqrt{1 + \frac{2\Psi}{M^2}}}, \tag{2.7}$$

$$n_{dp} = \sqrt{\frac{(M^2 + 3\alpha\sigma_{dp})}{6\alpha\sigma_{dp}}} \times \sqrt{\left(1 - \frac{2\alpha\Psi}{(M^2 + 3\alpha\sigma_{dp})}\right) - \sqrt{\left(1 - \frac{2\alpha\Psi}{(M^2 + 3\alpha\sigma_{dp})}\right)^2 - \frac{12\alpha\sigma_{\mathrm{dp}}M^2}{(M^2 + 3\alpha\sigma_{dp})^2}}}. \tag{2.8}$$

If we neglect the pressure of the positive dust fluid ($\sigma_{dp} = 0$), instead of (2.8) we have

$$n_{dp} = \frac{1}{\sqrt{1 - \frac{2\alpha\Psi}{M^2}}}. \tag{2.9}$$

The expressions (2.7)–(2.9) are identical to those obtained in [30]. Now, substituting for the densities in Poisson's equation (transformed to the co-moving frame in which the nonlinear structures are stationary), and after some algebraic manipulation, we finally obtain the energy integral form $\frac{1}{2}(\frac{\mathrm{d}\Psi}{\mathrm{d}\xi})^2 + V(\Psi) = 0$ where the expression for the Sagdeev potential $V(\psi)$ is given by

$$
\begin{aligned}
V(\Psi) &\\
&= M^2\left\{1-\sqrt{1+\frac{2\Psi}{M^2}}\right\}+\mu_i(1+3\beta)-\mu_i(1+3\beta+3\beta\Psi+\beta\Psi^2)\exp(-\Psi)\\
&+\frac{\mu_e}{\sigma}\{1-\exp(\sigma\Psi)\}+\frac{\mu_{dp}M\sqrt{(M^2+3\alpha\sigma_{\mathrm{dp}})}}{\alpha\sqrt{2}}\left\{\sqrt{1+\sqrt{1-\frac{12\alpha\sigma_{\mathrm{dp}}M^2}{(M^2+3\alpha\sigma_{\mathrm{dp}})^2}}}\right.\\
&\left.-\sqrt{\left(1-\frac{2\alpha\Psi}{(M^2+3\alpha\sigma_{dp})}\right)+\sqrt{\left(1-\frac{2\alpha\Psi}{(M^2+3\alpha\sigma_{dp})}\right)^2-\frac{12\alpha\sigma_{\mathrm{dp}}M^2}{(M^2+3\alpha\sigma_{dp})^2}}}\right\}\\
&+\frac{2\sqrt{2}\sigma_{\mathrm{dp}}\mu_{dp}M^3}{(\sqrt{M^2+3\alpha\sigma_{\mathrm{dp}}})^3}\left\{\left(\sqrt{1+\sqrt{1-\frac{12\alpha\sigma_{\mathrm{dp}}M^2}{(M^2+3\alpha\sigma_{\mathrm{dp}})^2}}}\right)^{-3}\right.\\
&\left.-\left(\sqrt{\left(1-\frac{2\alpha\Psi}{(M^2+3\alpha\sigma_{dp})}\right)+\sqrt{\left(1-\frac{2\alpha\Psi}{(M^2+3\alpha\sigma_{dp})}\right)^2-\frac{12\alpha\sigma_{dp}M^2}{(M^2+3\alpha\sigma_{dp})^2}}}\right)^{-3}\right\}.
\end{aligned}
\tag{2.10}
$$

If we neglect non-thermal effects of the ions by setting $\beta=0$, our expression (2.10) for the Sagdeev potential reduces to that of Mamun [30].

The solitary wave solutions exist if (i) $(d^2V/d\Psi^2)_{\Psi=0}<0$ so that a maximum occurs at the origin (the fixed point at the origin is unstable); (ii) $V(\Psi)<0$ when $0<\Psi<\Psi_{\max}$ for positive potential solitary waves and $\Psi_{\min}<\Psi<0$ for negative potential solitary waves where $\Psi_{\max(\min)}$ is the maximum (minimum) value of Ψ for which $V(\Psi)=0$; and (iii)$(d^3V/d\Psi^3)_{\Psi=0}>0$ (<0) for positive (negative) potential solitary waves. In addition to the local maximum condition at the origin, one requires $V(\Psi=\Psi_m)=0$, $(dV(\Psi)/d\Psi)_{\Psi=\Psi_m}=0$ and $(d^2V/d\Psi^2)_{\Psi=\Psi_m}<0$ such that $V(\Psi)<0$ for $0<\mid\Psi\mid<\mid\Psi_m\mid$ for negative or positive potential double-layer solutions.

In the limit of small solitary wave amplitude, one may expand $V(\Psi)$ about $\Psi=0$ to third order to obtain

$$V(\Psi)=C_2\Psi^2+C_3\Psi^3, \tag{2.11}$$

where

$$
\begin{aligned}
C_2 &= \frac{1}{2}(\mathrm{d}^2V(\Psi)/\mathrm{d}\Psi^2)_{\Psi=0}\\
&= -\frac{1}{2}\left[\mu_i(1-\beta)+\mu_e\sigma-\frac{1}{M^2}\left(1+\frac{\alpha\mu_{dp}}{[1-(3\alpha\sigma_{\mathrm{dp}}/M^2)]}\right)\right]
\end{aligned}
\tag{2.12}
$$

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and

$$C_3 = \frac{1}{6}(\mathrm{d}^3V(\Psi)/\mathrm{d}\Psi^3)_{\Psi=0}$$

$$= \frac{1}{6}\left[\mu_i - \mu_e\sigma^2 - \frac{3}{M^4}\left(1 - \alpha^2\mu_{dp}\frac{(1+(\alpha\sigma_{\mathrm{dp}}/M^2))}{[1-(3\alpha\sigma_{\mathrm{dp}}/M^2)]^3}\right)\right]. \quad (2.13)$$

We note that in the absence of non-thermal ion effects ($\beta = 0$), our derived expressions for C_2 and C_3 agree with those of Mamun [30].

The solution of (2.11) is given by Mamun [30] as

$$\Psi = \left(-\frac{C_2}{C_3}\right)\mathrm{sech}^2\left(\sqrt{\frac{-C_2}{2}}\xi\right). \quad (2.14)$$

On examination of (2.14), we can conclude that for $C_2 < 0$ (local maximum condition at origin) that small amplitude positive (negative) potential solitary wave solutions occur if $C_3 > 0$ ($C_3 < 0$).

In order for the local maximum condition to be satisfied at the origin, viz. $C_2 < 0$, where C_2 is given by (2.12), one requires $M > M_{\mathrm{crit}}$, where M_{crit} is obtained from

$$M_{\mathrm{crit}} = \sqrt{\frac{1+\mu_{\mathrm{dp}}\alpha + 3\alpha\sigma_{\mathrm{dp}}[\mu_i - \mu_i\beta + \mu_e\sigma]}{2(\mu_i - \mu_i\beta + \mu_e\sigma)}} \times \left\{1 + \sqrt{1 - \frac{12\alpha\sigma_{\mathrm{dp}}(\mu_i - \mu_i\beta + \mu_e\sigma)}{[1+\mu_{\mathrm{dp}}\alpha + 3\alpha\sigma_{\mathrm{dp}}(\mu_i - \mu_i\beta + \mu_e\sigma)]^2}}\right\}^{1/2}. \quad (2.15)$$

3. Numerical results and discussion

Figure 1 depicts the small amplitude results. One of the requirements for a solitary wave solution is that $V(\Psi)$ given by (2.10) must have a local maximum at the origin, i.e. the Mach number must exceed the value M_{crit} obtained from the expression (2.15). An examination of (2.14) reveals that the polarity of the potential of the small amplitude solitary wave that will occur is obtained from the sign of $C_3(M_{\mathrm{crit}})$, viz. a positive (negative) potential solitary wave will occur if $C_3(M_{\mathrm{crit}}) > 0$ ($C_3(M_{\mathrm{crit}}) < 0$), where $C_3(M_{\mathrm{crit}})$ is calculated from (2.13) with M replaced by the value M_{crit} obtained from (2.15). The fixed parameter α is the ratio of the charge-to-mass ratio of the positive dust to that of the negative dust, viz. $\alpha = (Z_{dp}/m_{dp})/(Z_{dn}/m_{dn})$. The variation of M_{crit} (Fig. 1a) and $C_3(M_{\mathrm{crit}})$ (Figs 1b and c) with α is shown for three different fixed values of the ion non-thermal parameter γ, viz. $\gamma = 0$, 0.1 and 0.2 and the same value for the temperature of the positive dust, viz. $\sigma_{dp} = 0.01$. Figure 1(c) clearly shows the region for which $C_3(M_{\mathrm{crit}}) < 0$ which is not clear in Fig. 1(b) due to the wideness of the range of α values for which $C_3(M_{\mathrm{crit}})$ is depicted.

We now use our obtained expression (2.15) for the critical Mach number and (2.13) which confirms the polarity of the potential of the small amplitude solitary wave to investigate the existence of large amplitude solitary waves by examining plots of the Sagdeev potential $V(\Psi)$ given by (2.10) for different combinations of values for the plasma parameters. For the same fixed value of the magnitude of the ion non-thermal parameter, viz. $\gamma = 0.1$ and for $\sigma_{dp} = 0.01$, we select three different values of α, viz. $\alpha = 1$, 10 and 100, where all other parameters are fixed

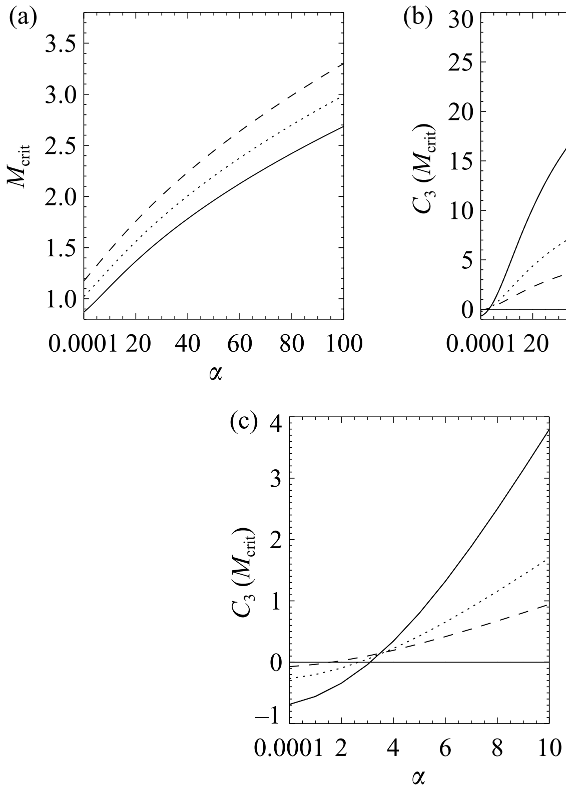

Figure 1. (a) The critical value of the Mach number as a function of $\alpha = Z_{dp}m_{dn}/(Z_{dn}m_{dp})$ required for the existence of positive (negative) potential solitary waves if $C_3(M_{\rm crit}) > 0$ ($C_3(M_{\rm crit}) < 0$) shown in (b) and (c). The curves correspond to $\gamma = 0$ (–), $\gamma = 0.1$ ($\cdots$) and $\gamma = 0.2$ (- -). The fixed parameters are $\sigma_{dp} = 0.01$, $\sigma = T_i/T_e = 0.5$, $\mu_e = 0.25$, $\mu_i = 1.2$ and $\mu_{dp} = 1 + \mu_e - \mu_i = 0.05$.

as indicated in Fig. 1 and investigate the existence of large amplitude solitary waves. For $\alpha = 1$, $\gamma = 0.1$ and the other fixed parameters indicated in Fig. 1, plots of (2.10) shown in Fig. 2 for different values of the Mach number, reveal that only *negative* potential solitary waves (NPSWs) occur when M exceeds the value $M_{\rm crit} = 1.04884$ (Fig. 1a). The *negative* polarity for the electrostatic potential of the solitary wave is confirmed by examining Fig. 1(c), where it can be seen for $\alpha = 1$ and $\gamma = 0.1$, $C_3(M_{\rm crit} = 1.04884) < 0$. For $\alpha = 100$ and $\gamma = 0.1$, the Sagdeev potential profiles depicted in Fig. 3 reveal that only *positive* potential solitary waves (PPSWs) occur when $M > M_{\rm crit} = 2.98764$. In this case, the *positive* polarity for the potential is confirmed by Fig. 1(b), where it is observed that $C_3(M_{\rm crit} = 2.98764) > 0$ for $\alpha = 100$. Now for an intermediate choice of the magnitude of α, viz. $\alpha = 10$ and $\gamma = 0.1$, Fig. 1 predicts that when $M > M_{\rm crit}$ where $M_{\rm crit} = 1.29727$, a small amplitude solitary wave having *positive* potential will occur because $C_3(M_{\rm crit} = 1.29727) > 0$ (more clear in Fig. 1c). However, our large amplitude solitary wave results depicted for $\alpha = 10$ in Fig. 4 reveal that solitary waves of both *positive* and *negative* potentials occur when M exceeds the value 1.29727. The limitations of using the small wave amplitude theory to investigate the existence of arbitrary amplitude

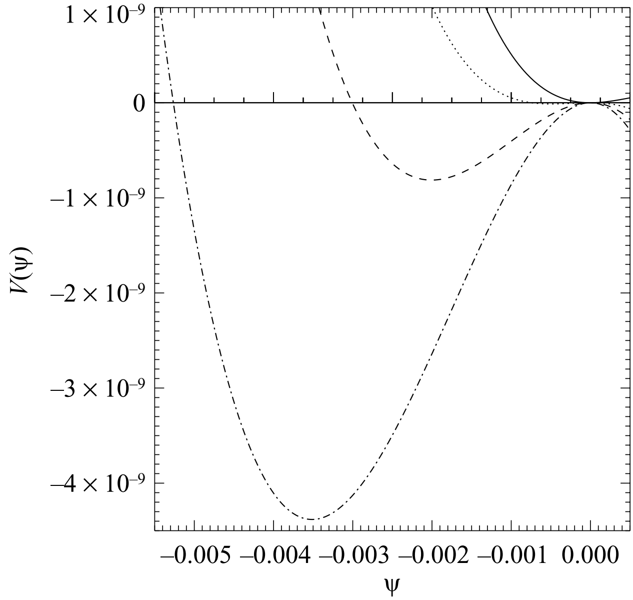

Figure 2. The Sagdeev potential as a function of Ψ for negative potentials. The curves correspond to $M = 1.0485$ (–), $M = 1.049$ ($\cdots$), $M = 1.0495$ (- -) and $M = 1.05$ (— · —). The fixed parameters are $\alpha = Z_{dp}m_{dn}/(Z_{dn}m_{dp}) = 1$, $\gamma = 0.1$, $\sigma = T_i/T_e = 0.5$, $\mu_e = 0.25$, $\mu_i = 1.2$, $\mu_{dp} = 1 + \mu_e - \mu_i = 0.05$ and $\sigma_{dp} = 0.01$.

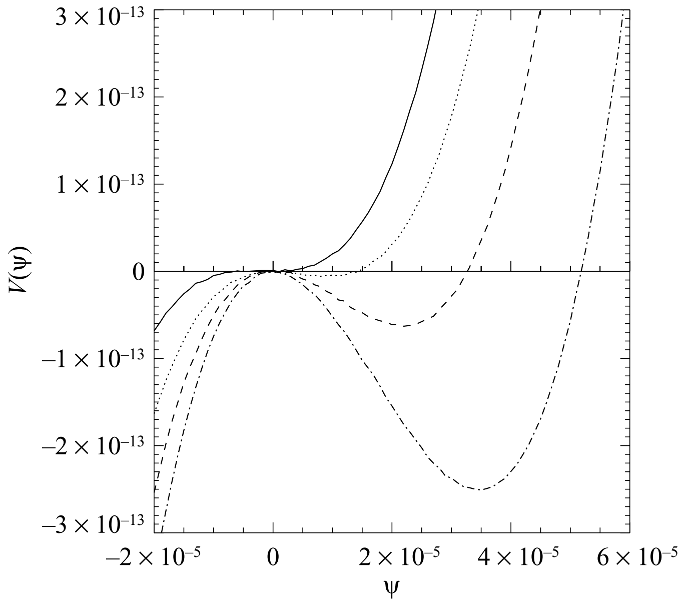

Figure 3. The Sagdeev potential as a function of Ψ for positive potentials. The curves correspond to $M = 2.9875$ (–), $M = 2.988$ ($\cdots$), $M = 2.9885$ (- -) and $M = 2.989$ (—·—). The fixed parameters are $\alpha = Z_{dp}m_{dn}/(Z_{dn}m_{dp}) = 100$, $\gamma = 0.1$, $\sigma = T_i/T_e = 0.5$, $\mu_e = 0.25$, $\mu_i = 1.2$, $\mu_{dp} = 1 + \mu_e - \mu_i = 0.05$ and $\sigma_{dp} = 0.01$.

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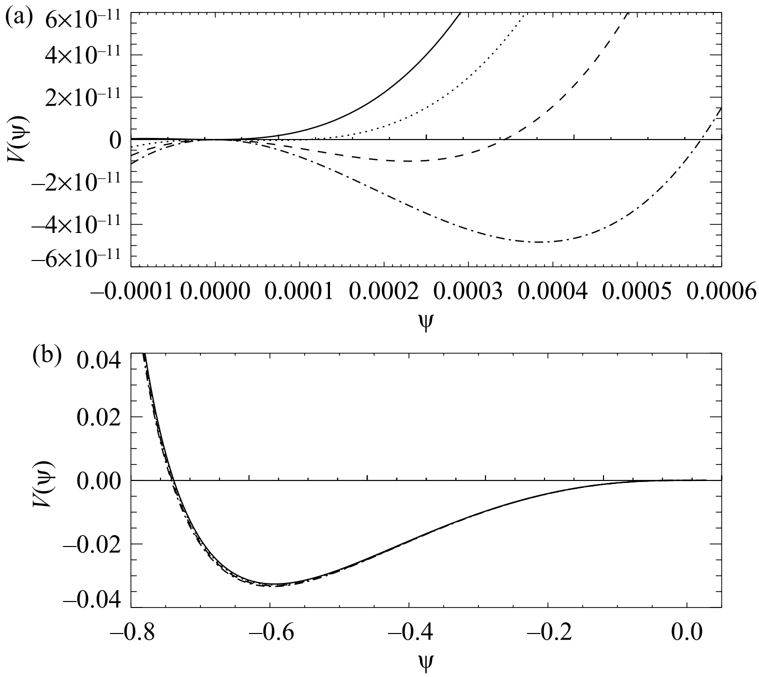

Figure 4. The Sagdeev potential as a function of Ψ. The curves in (a) and (b) correspond to $M = 1.297$ (–), $M = 1.2975$ $(\cdots)$, $M = 1.298$ (- -) and $M = 1.2985$ $(- \cdot -)$. The fixed parameters are $\alpha = Z_{dp}m_{dn}/(Z_{dn}m_{dp}) = 10$, $\gamma = 0.1$, $\sigma = T_i/T_e = 0.5$, $\mu_e = 0.25$, $\mu_i = 1.2$, $\mu_{dp} = 1 + \mu_e - \mu_i = 0.05$ and $\sigma_{dp} = 0.01$.

solitary waves thus become apparent when the coexistence of PPSWs and NPSWs of arbitrary amplitude occurs for the same set of fixed parameters, because the small amplitude theory predicts the existence of a single solitary wave (of either positive or negative polarity). To summarize, our large amplitude solitary wave results depicted by plots of (2.10) for different values of the Mach number reveal the existence of only NPSWs for $\alpha = 1$ $(1.0485 < M \leq 1.05)$ (Fig. 2), the existence of only PPSWs for $\alpha = 100$ $(2.9875 < M \leq 2.989)$ (Fig. 3) but the coexistence of both NPSWs and PPSWs for $\alpha = 10$ $(1.297 < M \leq 1.2985)$ (Fig. 4) for a plasma with non-thermal ions $(\gamma = 0.1)$ and for the other fixed parameters as indicated for Fig. 1. The important observation in each of Figs 2–4 is that the solitary waves are seen to get stronger with increasing values of M; however, here, we did not determine upper limiting values of the Mach number, which restrict the solitary wave existence domains from the high-Mach-number end.

4. Conclusions

We have investigated the existence of large amplitude solitary waves for a model composed of a cold negative dust fluid, adiabatic positive dust, Boltzmann electrons and non-thermal ions. This model is a generalization of the model considered in [30] in which a non-thermal velocity distribution is adopted for the ions. The non-thermal distribution that we utilize for the ions is similar to the distribution used

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by Cairns et al. [17] to describe non-thermal electrons and is characterized by a non-thermal parameter γ. The number density for the non-thermal ions reduces to the case of Boltzmann ions considered in [30] for $\gamma = 0$. Three different existence regimes for large amplitude solitary waves have been identified, viz. the existence of only NPSWs for $\alpha = 1$ ($1.0485 < M \leq 1.05$), the existence of only PPSWs for $\alpha = 100$ ($2.9875 < M \leq 2.989$), but the coexistence of NPSWs and PPSWs for $\alpha = 10$ ($1.297 < M \leq 1.2985$), where $\alpha = (Z_{dp}/m_{dp})/(Z_{dn}/m_{dn})$ is a ratio of the charge-to-mass ratios of the positive and negative dust grains. Our choice of values of the Mach number for which solitary waves occur is well within the permitted Mach number ranges; however, it is important to mention the existence of upper limiting values of the Mach number that restrict the solitary wave existence domains from the high-Mach-number end. The problem of existence domains of large amplitude solitary structures in dusty plasmas is currently being investigated, and the results will be reported elsewhere.

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