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Do nonlinear waves evolve in a universal manner in dusty and other plasma environments?

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Using a fluid theory approach, this article provides a comparative study on the evolution of nonlinear waves in dusty plasmas, as well as other plasma environments, viz electron-ion, and electron-positron plasmas. Where applicable, relevance to satellite measurements is pointed out. A range of nonlinear waves from low frequency (ion acoustic and ion cyclotron waves), high frequency (electron acoustic and electron cyclotron waves) in electron-ion plasmas, ultra-low frequency (dust acoustic and dust cyclotron waves) in dusty plasmas and in electron-positron plasmas are discussed. Depending upon the plasma parameters, saw-tooth and bipolar structures are shown to evolve.

1. Introduction

In the last two decades, the interest of the scientific community in the field of dusty plasma research has grown manyfold. This has been possible due to importance of dust and their frequent occurrence in space and astrophysical environments, e.g. cometary tails, planetary rings and magnetospheres, noctilucent clouds in the lower atmosphere of the Earth and laboratory plasmas. Dusty plasmas are unique in the sense that a dust grain is very heavy and acquires a large negative or positive charge. The motion of charged dust grains supports ultra-low frequency waves such as dust-acoustic waves (DAWs). In most of the studies on nonlinear waves in dusty and electron-ion plasmas, emphasis has been given to solitons and double layers and only few studies have been carried on driven nonlinear electric field structures (Lee and Kan 1981). For example, Reddy et al. (2002, 2006), Bharuthram et al. (2002), Moolla et al. (2003, 2007) have studied driven low (ion-acoustic) and high (electron-acoustic) frequency waves in multi-component electron-ion plasmas. These studies were prompted by the FAST and Geotail satellite observations of spiky structures in the parallel electric field in the downward current region of auroral magnetosphere (Mozer et al. 1997, Ergun et al. 1998) and in the plasma sheet boundary layer (Kojima et al. 1994). Theoretical models to explain the spiky electric field structures were given by Reddy et al. (2002). It was shown, for example, when the amplitude of the driver electric field increased, the wave evolved from linear sinusoidal to sawtooth to a

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spiky bipolar structure. Subsequently, studies were undertaken in different plasma environments. For example, Maharaj et al. (2008) studied the nonlinear evolution of driven low-frequency electrostatic waves in a three-component magnetized dusty plasma comprised of a warm dust fluid, electrons and ions. The periodic solutions of the nonlinear differential equation yielded sinusoidal, sawtooth and bipolar structures similar to the nonlinear structures supported in electron-ion plasmas. Verheest (2009) studied the oblique propagation of solitary electrostatic waves in multispecies plasmas and showed the nonlinear equations to fit in a general and compact framework.

In this article, we review the evolution of nonlinear waves in dusty plasmas, as well as other plasma environments, viz electron-ion, and also present recent findings of spiky structures in electron-positron plasmas. The results appear to imply that the nonlinear structures evolve in a universal manner in dusty and other plasma environments.

2. Theoretical models and nonlinear analysis

For a start, we consider a three-component, homogeneous, magnetized dusty plasma having negatively charged dust particles, electrons and protons. Further, electrons and ions are assumed to be Boltzmann distributed, negatively charged dust particles are drifting along the ambient magnetic field, $\mathbf{B_0}$ and waves are propagating in the $x-$ direction at an angle θ to the $\mathbf{B_0}$ which is in the x-z plane. The basic set of governing fluid equations for such a dusty plasma model can be written as (Maharaj et al. 2008)

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_{dx}) = 0 \tag{2.1}$$

$$\frac{\partial v_{dx}}{\partial t} + v_{dx}\frac{\partial v_{dx}}{\partial x} + \frac{1}{n_d m_d}\frac{\partial p_d}{\partial x} = \frac{Z_d e}{m_d}\frac{\partial \phi}{\partial x} - \Omega_{\text{cd}}\, v_{dy} \sin\theta \tag{2.2}$$

$$\frac{\partial v_{dy}}{\partial t} + v_{dx}\frac{\partial v_{dy}}{\partial x} = \Omega_{\text{cd}}\, v_{dx} \sin\theta - \Omega_{\text{cd}}\, v_{dz} \cos\theta \tag{2.3}$$

$$\frac{\partial v_{dz}}{\partial t} + v_{dx}\frac{\partial v_{dz}}{\partial x} - \Omega_{\text{cd}}\, v_{dy} \cos\theta = 0, \quad \frac{\partial p_d}{\partial t} + v_{dx}\frac{\partial p_d}{\partial x} + 3p_d\frac{\partial v_{dx}}{\partial x} = 0 \tag{2.4}$$

$$n_{\text{e}} = n_{e0} \exp\left(\frac{e\phi}{T_e}\right), \quad n_{\text{i}} = n_{i0} \exp\left(-\frac{e\phi}{T_{\text{i}}}\right) \tag{2.5}$$

where ϕ is the electrostatic wave potential and p_d is the dust plasma pressure, n_j is the number density of species $j (j = d, e, p$ refers to dust, electrons and protons, respectively), subscript $'0'$ refers to equilibrium quantities and $\Omega_{\text{cd}} = (Z_d e B_0 / m_d c)$ is the dust gyrofrequency; c is the speed of light, Z_d and m_d are the dust charge and mass, respectively and T_j is the temperature of species j. Further, v_{dx} v_{dy} and v_{dz} are the $x-$, $y-$ and $z-$components of the dust fluid velocity. The charge neutrality condition at equilibrium is $n_0 = n_{i0} = n_{e0} + Z_d n_{d0}$ with n_0 being the total ion density. We seek the solution of (2.1)–(2.5) for nonlinear analysis and transform these equations to a stationary frame through variable $\eta = (x - Vt)/(\Omega_{\text{cd}}/V)$ where V is the phase speed of the wave. Before proceeding with the analysis, it should be noted that henceforth, all equations are written in the normalized form. Further, we have used the following normalizations: velocities are normalized with respect to effective dust-acoustic speed $c_{sd} = \sqrt{T_{eff}/m_d}$ ($T_{\text{eff}} = Z_d^2 n_{d0}/(n_{e0}/T_e + n_{i0}/T_i)$; electrostatic wave potential ϕ is normalized with $(T_{\text{eff}}/Z_d e)$ and dust plasma pressure p_d is normalized with $(n_{d0} T_d)$,

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n_j is number density of species $j (j = i, e)$ normalized with $(Z_d n_{d0})$ and n_{d} is the number density of dust particles normalized with respect to its equilibrium density n_{d0}. The distance is normalized with effective Larmor radius $\rho_{\mathrm{sd}} = (c_{\mathrm{sd}}/\Omega_{\mathrm{cd}})$; and time is normalized with $\Omega_{\mathrm{cd}}^{-1}$.

Thus, the number densities of the electrons and ions can be written as

$$n_{\mathrm{e}} = \delta'_e \exp(\alpha\Phi), \quad n_{\mathrm{i}} = \delta'_i \exp\left(-\frac{\alpha\Phi}{\beta}\right) \tag{2.6}$$

where $\beta = T_i/T_e$, $\delta'_i = n_{i0}/Z_d n_{d0}$ and $\delta'_e = n_{e0}/Z_d n_{d0}$, $\Phi = Z_d e\phi/T_{\mathrm{eff}}$ and $\alpha = T_{\mathrm{eff}}/Z_d T_e = \beta\delta_d/(1 + \beta\delta_e)$; $\delta_{\mathrm{d}} = Z_d n_{\mathrm{d}0}/n_0$ and $\delta_{\mathrm{e}} = n_{\mathrm{e}0}/n_0$ are related via the quasi-neutrality condition $1 = \delta_e + \delta_d$. Since, low-frequency, long wavelength electrostatic waves are being studied here, we assume the quasi-neutrality condition for the perturbed quantities, given by

$$n_{\mathrm{d}} = n_i - n_{\mathrm{e}} = \delta'_i \mathrm{e}^{-\alpha\Phi/\beta} - \delta'_e \mathrm{e}^{\alpha\Phi} \tag{2.7}$$

Using the boundary conditions $n_d = 1$ and $v_{dx} = v_{d0}\cos\theta, v_{dy} = 0$ and $v_{dz} = v_{d0}\sin\theta$, at $\eta = 0$, integration of (2.1) and alongwith (2.7) yields

$$v_{dx} - M = \frac{-(M - \Lambda)}{\delta_i{}'\mathrm{e}^{-\alpha\Phi/\beta} - \delta_e{}'\mathrm{e}^{\alpha\Phi}} = L_x \tag{2.8}$$

where we have defined $\Lambda = v_{d0}\cos\theta$ and $M = V/c_{\mathrm{sd}}$ is the Mach number. In the stationary frame the normalized set of (2.2)–(2.4), yield

$$L_x\frac{\mathrm{d}v_{dx}}{\mathrm{d}\eta} + \frac{\sigma_d}{n_d}\frac{\mathrm{d}p_d}{\mathrm{d}\eta} = \frac{\mathrm{d}\Phi}{\mathrm{d}\eta} - Mv_{dy}\sin\theta, \quad L_x\frac{\mathrm{d}v_{dy}}{\mathrm{d}\eta} = Mv_{dx}\sin\theta - Mv_{dz}\cos\theta \tag{2.9}$$

$$L_x\frac{\mathrm{d}v_{dz}}{\mathrm{d}\eta} = Mv_{dy}\cos\theta, \quad L_x\frac{\mathrm{d}p_d}{\mathrm{d}\eta} + 3p_d\frac{\mathrm{d}v_{dx}}{\mathrm{d}\eta} = 0 \tag{2.10}$$

We obtain a single differential equation after eliminating velocity components v_{dx}, v_{dy} and v_{dz} and dust number density n_d from (2.9)–(2.10) which can be written as

$$\mathrm{A}(\Phi)\frac{\mathrm{d}^2\Phi}{\mathrm{d}\eta^2} + \mathrm{B}(\Phi)\left(\frac{\mathrm{d}\Phi}{\mathrm{d}\eta}\right)^2 + \mathrm{C}(\Phi) = 0 \tag{2.11}$$

where we have used the boundary conditions $\Phi = 0$, $\mathrm{d}^2\Phi/\mathrm{d}\eta^2 = 0$ and $\mathrm{d}\Phi/\mathrm{d}\eta = E_0$ at $\eta = 0$. Since, we are studying the driven modes, here E_0 is the driving electric field which comes out due to the integration of (2.9) through the boundary condition $\mathrm{d}\Phi/\mathrm{d}\eta = E_0$ at $\eta = 0$ (Lee and Kan 1981). The coefficients $\mathrm{A}(\Phi)$, $\mathrm{B}(\Phi)$ and $\mathrm{C}(\Phi)$ are given in Maharaj et al. (2008). It is interesting to note that (2.11) describes the nonlinear evolution of the coupled dust-acoustic and dust-cyclotron modes in dusty plasmas. It is also important to note that it is similar to the equation obtained by Reddy et al. (2002, 2006) for the coupling of the low-frequency ion-acoustic and ion cyclotron waves in electron-ion plasmas. In the next section, we show the numerical solutions of (2.11) for various fixed values of plasma parameters.

3. Numerical results

In Figs. 1(a)–(d), the parallel electric field $(\mathrm{d}\Phi/\mathrm{d}\eta)$ versus η is shown for different values of the driving electric field amplitude E_0 for a dusty plasma model with the results taken from Maharaj et al. (2008). The other parameters are: $\theta = 2^\circ$, $M = 1.25$, $v_{d0} = 0$, $\sigma_{\mathrm{d}} = T_d/T_{\mathrm{eff}} = 0$, $\delta_e = n_{e0}/n_0 = 0.75$, $\beta = T_i/T_e = 0.1$. It is observed

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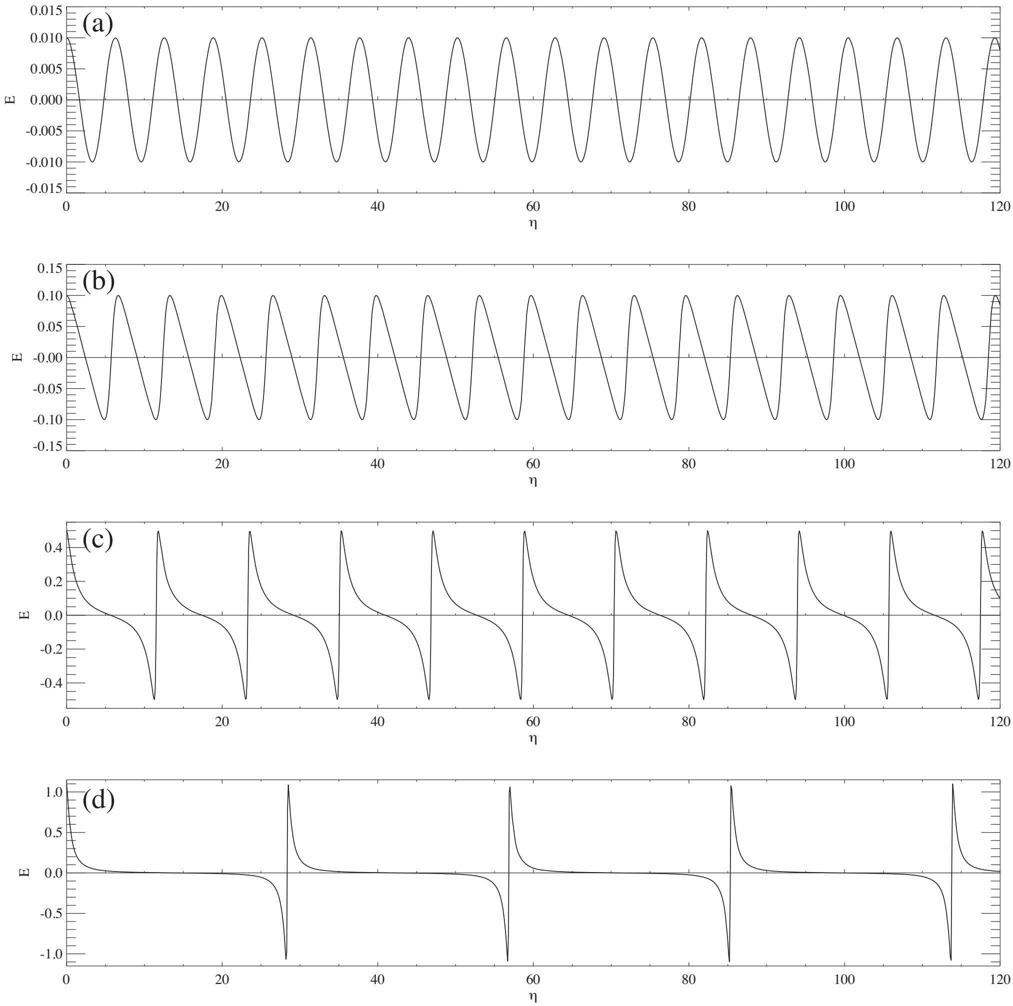

FIGURE 1. The normalized parallel electric field ($= \mathrm{d}\Phi/\mathrm{d}\eta$) of nonlinear electrostatic dust cyclotron and dust acoustic waves for the parameters $\theta = 2°$, $M = 1.25$, $v_{d0} = 0$, $\sigma_{\rm d} = T_d/T_{\rm eff} = 0$, $\delta_e = n_{e0}/n_0 = 0.75$, $\beta = T_i/T_e = 0.1$ and E_0=0.01(a), 0.1(b), 0.5(c) and 1.1(d) (*taken from Maharaj et al. 2008*).

that as the driving field, E_0 increases from small ($E_0 = 0.01$) to large ($E_0 = 1.1$), the electric field structures changes from sinusoidal to highly spiky and period of oscillation increases. These sinusoidal structures with small periods are interpreted as dust-cyclotron modes since the period, $\tau = 2\pi/\Omega_{cd}$ and the lower frequency highly spiky structures are the driven dust-acoustic waves. One can see the transition from dust-cyclotron to dust-acoustic waves with increasing magnitude of the driver electric field.

We now focus our attention to electron-ion plasmas described by a similar set of fluid equations as described by Reddy et al. (2006) and repeated here from their work. In Fig. 2, the variation of the parallel electric field for for M = 1.25 and for different electric field driver strengths, E_0, for a homogeneous magnetized three component plasma consisting of electrons, protons and cold oxygen ion beam drifting along the magnetic field is shown. The chosen parameters for this study are $\theta = 2^o$, M=1.25,

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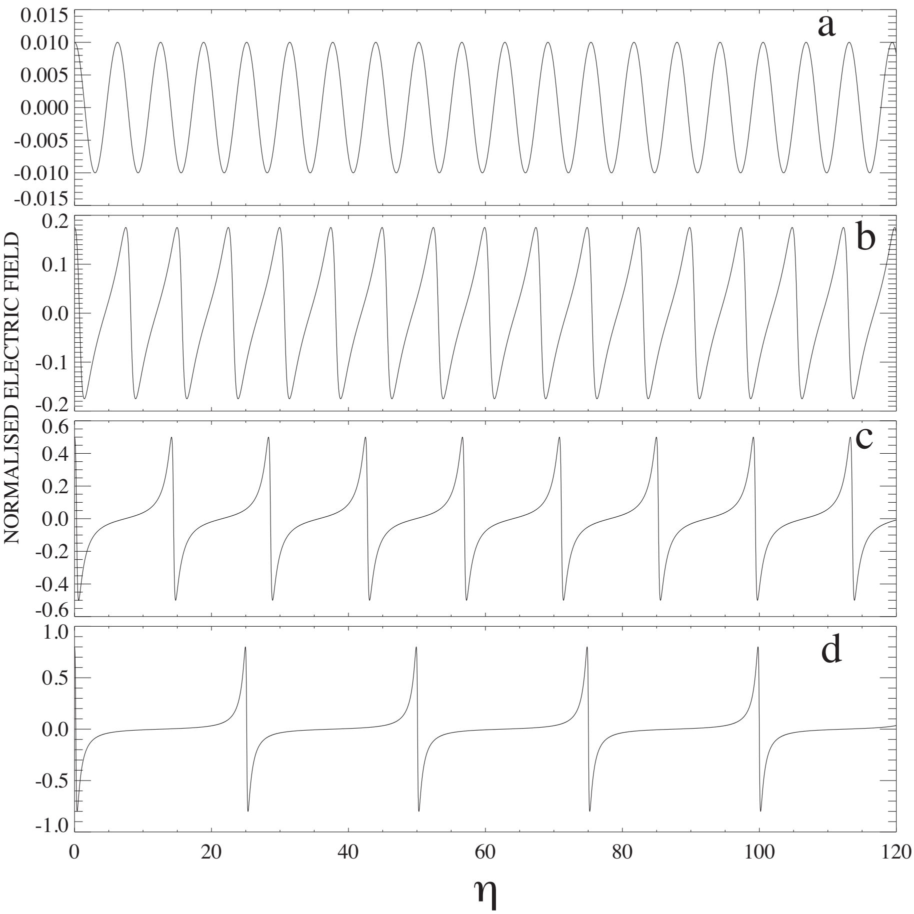

FIGURE 2. The normalized parallel electric field of nonlinear electrostatic ion cyclotron and ion acoustic waves for the parameters $\theta = 2^o$, M=1.25, $v_0 = 0$, $N_{i0} = 0.75$, $T_e/T_p = 0.1$ and $E_0 = 0.01$(a), 0.175(b), 0.50(c), and 0.80(d) (*taken from Reddy et al. 2006*).

$v_0 = 0$, $N_{i0} = 0.75$, $T_e/T_p = 0.1$. Here for small $E_0 = 0.01$(Fig. 2(a)), the sinusoidal oscillations with a period equal to that of the ion cyclotron wave are observed. Further, as the driver strength is increased, the oscillations become nonlinear with increasing periodicity and at large driver amplitude (E_0=0.8), the driven ion acoustic waves with highly spiky electric field are obtained. This behaviour is consistent with the measurements of parallel electric field by the FAST satellite (Fig. 6 of Ergun et al. 1998).

In the present study, we also examine the evolution of the electric field structures in a homogeneous magnetized, four component, collisionless, electron-positron plasma, consisting of cool electrons and positrons with equal temperatures and hot electrons and positrons with equal temperatures. The nonlinear coupling occurs between cyclotron and acoustic modes in the electron-positron plasmas. Figures 3(a)–(d) show the evolution of the electric field structures for various driving electric field amplitudes E_0 for the fixed parameters: $M = 3.5$, $R = \omega_p/\Omega = 10.0$, $\theta = 2^o$, $\delta_c = \delta_h = 0.0$, $n_{ec0}/n_0 = n_{pc0}/n_0 = 0.5$ and $T_c/T_h = 0.0$; n_{ec0}, n_{pc0} refers to cool electron and positron densities respectively. Here $\omega_p = \sqrt{4\pi n_0 e^2/m}$; n_0 being total electron density and m

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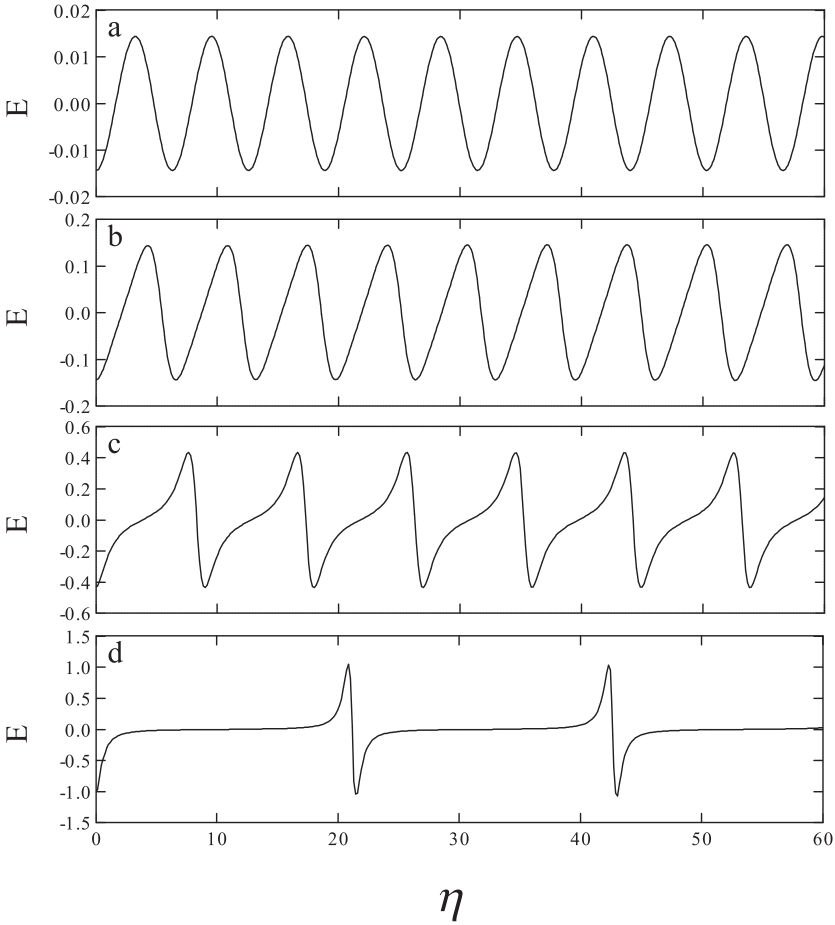

FIGURE 3. Numerical solution of the normalized electric field for an electron-positron plasma for the parameters $M = 3.5$, $\theta = 2^o$, $R = 10.0$, $\delta_c = \delta_h = 0.0$, $n_{0c}/n_0 = 0.5$, $T_c/T_h = 0.0$, and E_0= 0.014 (a), 0.14 (b), 0.43 (c) and 1.0 (d).

is the common mass of the electrons and protons and Ω are the electron (positron) plasma and cyclotron frequencies, respectively. Note that as in the earlier studies wave propagation is taken almost parallel to the ambient magnetic field $\mathbf{B_0}$. As in earlier studies, the electric field structure evolves from a sinusoidal wave corresponding to the cyclotron mode to a sawtooth structure and finally to spiky bipolar as E_0 increases. It is observed that the period of the spiky structure is about three and a half times the cyclotron period and hence we deduce that the waveform is the driven acoustic mode (Lazarus et al. 2014).

A similar study on high frequency nonlinear waves has been conducted by Moolla et al. (2007) in a magnetized three component plasma consisting of hot electrons, cool electrons, and cool ions. They have numerically solved the nonlinear system of governing equations. The coupling of high frequency electron-acoustic and electron cyclotron mode occurs. Their study also showed the transition from sinusoidal electron cyclotron waveforms for low driving amplitude to highly spiky structures corresponding to electron-acoustic mode for larger values of the driving amplitude (cf Fig. 1(a)–(d), Moolla et al. 2007) . It was also shown that the drifts of the species play a crucial role in controlling the periods of the waveforms.

4. Conclusions

In this paper we have reviewed the evolution of nonlinear electric field structures in different plasma environments such as dusty plasmas, multi-component electron-ion plasmas and electron-positron plasmas. Using a fluid theory approach, we have reviewed studies on the ultra low frequency (dust acoustic and dust cyclotron waves), low frequency (ion acoustic and ion cyclotron waves) and high frequency (electron acoustic and electron cyclotron waves) nonlinear waves in magnetised plasmas through coupling between respective acoustic and cyclotron modes. In addition, a review of similar modes in an electron-positron plasma is considered. Waves in all cases were considered to be propagating almost parallel ($\theta = 2^0$) to the ambient magnetic field. The nonlinear electric field structures showed similar behaviour in all plasma environments. Their evolution started from sinusoidal to sawtooth to highly spiky structures when the driver electric field increased from low to high values. The sinusoidal waves showed periods matching those of the corresponding cyclotron waves whereas the spiky electric field structure periods matched the acoustic modes. Therefore, it can be concluded that these highly spiky nonlinear structures evolve in similar manner irrespective of the plasma environment.

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